At a certain density cars stand still. This maximum density, ρ_{max}, usually corresponds to what is called bumper-to-bumper traffic,

$$\boxed{u(\rho_{max}) = 0.} \tag{61.5}$$

(Cars are observed to come to a stop in dense traffic before cars touch each other. Thus $\rho_{max} < 1/L$, where L is the average length of a vehicle.) Consequently, the general type of curve shown in Fig. 61-1, $u = u(\rho)$, relating the two traffic variables (velocity and density) is reasonable. As stated earlier, the curve is steadily decreasing, that is $u'(\rho) \leq 0$.

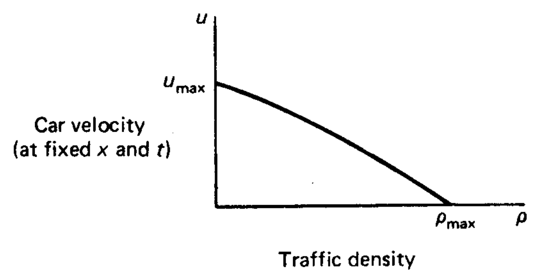

Figure 61-1 Car velocity diminishes as traffic density increases.

It is not being suggested that the dependence of the velocity on the density is the same for all road conditions; a different segment of the same highway might have a different relationship (for example, because of a different road curvature and banking). Furthermore there may be time dependence in the relationship, $u(\rho, x, t)$, as a result, for examples, of the effects a police car (either moving or stationary) or weather conditions have on car velocities. If a speed limit u_{SL} is obeyed, then, for example, $u_{max} = u_{SL}$. However, the speed limit need not be constant as it might differ in various sections of the roadway. (Usually, but not always, at fixed positions along the highway, the speed limit is the same. An exception occurs when the speed limit at night differs from that during the day. Another such example is reduced speed limits near some schools during school hours.) However, we will primarily investigate a given stretch of highway with approximately constant properties (same number of lanes, no intersections, and so on). Thus we will assume $u = u(\rho)$.

There are many assumptions involved in this hypothesis. It states that every driver drives at the same velocity given the same spacing (density). This is clearly not valid, though it may *not* be a very bad approximation. We ignore the possibly erratic behavior of individual drivers. Perhaps it would be more realistic to introduce a stochastic model in which it is proposed that at a certain density some percentage of drivers drive at certain speeds and others

drive at slightly different speeds. We will not discuss such stochastic models although some traffic researchers have developed mathematical models along these lines.

Some experiments have indicated that $u = u(\rho)$ is reasonable while traffic is accelerating and also reasonable while traffic is decelerating, but the velocity-density curve is different for those two circumstances.

If $u = u(\rho)$, then a high speed car as it approaches a slower line of traffic must itself slow down. This theory does not take into account that on multilane highways passing is not only permitted but is a quite frequent event. In order for this theory to be a good approximation, the effects of car passing must be small, as for examples, on one-lane roads, tunnels, or extremely crowded highways.

Another assumption is that the velocity only depends on the density, $u = u(\rho)$, not, for example, on the traffic density a few cars ahead (which may be quite different). Perhaps this is an important consideration because your own experience as a passenger or driver tells you to slow down if you observe trouble ahead; you do not wait to slow down until you actually reach the trouble. Modifications in the mathematical model to include these kinds of effects also have been attempted.

In addition $u = u(\rho, x, t)$ implies that if ρ changes, then u changes instantaneously. Thus the model as formulated will not take into account the finite driver reaction time nor the finite response time it takes an automobile engine to change velocity (accelerate or decelerate). These effects can also be incorporated into a more refined mathematical model.

Now that we have enumerated a number of objections to the model, let us nevertheless investigate its implications. This model represents a possible first step in the development of a mathematical theory of traffic flow.

EXERCISES

61.1. Many state laws say that for each 10 m.p.h. (16 k.p.h.) of speed you should stay at least one car length behind the car in front of you. Assuming that people obey this law (i.e., *exactly one* length), determine the density of cars as a function of speed (you may assume the average length of a car is 16 feet (5 meters)). There is another law that gives a maximum speed limit (assume this is 50 m.p.h. (80 k.p.h.)). Find the flow of cars as a function of density.

61.2. The state laws on following distances, briefly discussed in exercise 61.1, were developed in order to prescribe spacing between cars such that rear-end collisions could be avoided.

(a) Assume the car immediately ahead stops instantaneously. How far would the driver following at u m.p.h. travel, if