1 - 11 - 08

Note Title 1/11/2008

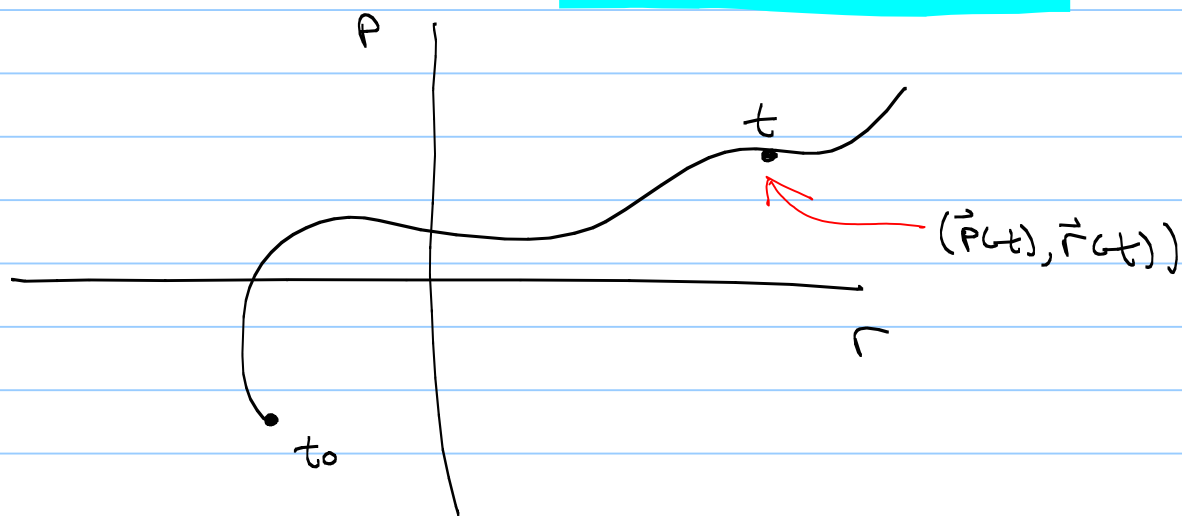
State of Classical particle completely specified by $\vec{r}(t)$. To get $\vec{r}(t)$ we solve $F = m\ddot{\vec{r}}$ subject to

$\vec{r}(0) = \vec{r}_0$ $\dot{\vec{r}}(0) = \vec{v}_0$.

Equivalently we can write $F = m\ddot{\vec{r}}$ as:

$$\dot{\vec{r}} = \vec{p}/m \qquad \vec{r}(0) = \vec{r}_0$$
$$\dot{\vec{p}} = \vec{F} \qquad \vec{p}(0) = \vec{p}_0$$

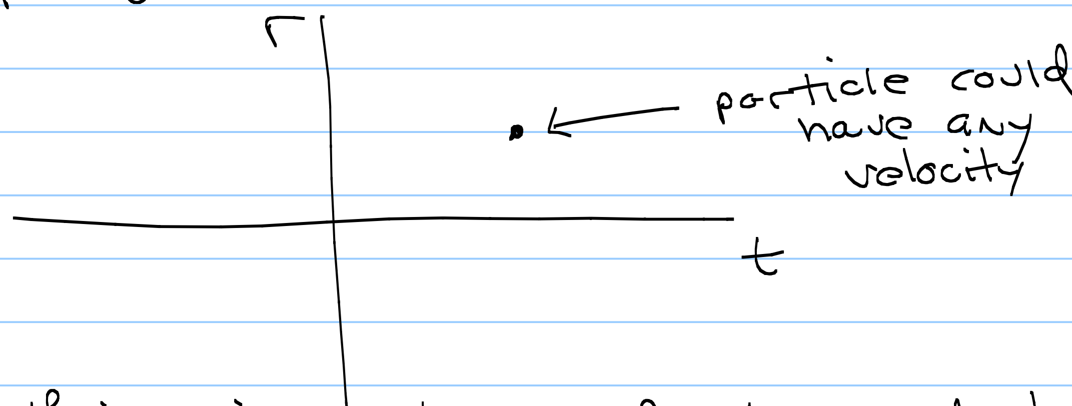
Now, $\vec{p}$ & $\dot{\vec{m}}$ are not independent since $\vec{p} = \dot{\vec{r}}\, m$. But we can plot $\vec{p}$ vs $\vec{r}$ parametrically



$\vec{r}$ - $\vec{p}$ space is called phase space

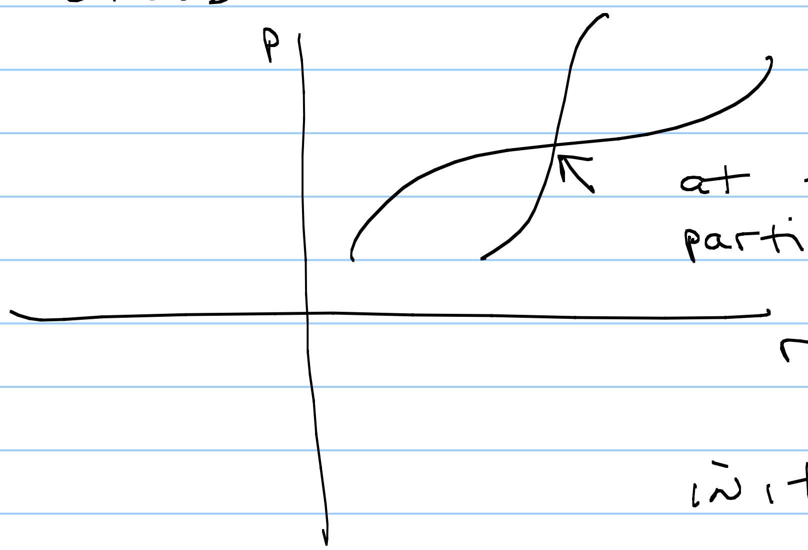
At any time t the state of the particle is completely specified by a point in phase space

where as a point on the graph of $\vec{r}$ vs t is not a complete specification of the state of the particle



So this is not enough to predict the evolution. But if we know $\vec{r}$ & $\vec{p}$ at a point we can.

Digression: A theorem from ODE's tells us that standard ODE's have unique solutions depending on initial conditions. This means that trajectories in phase space cannot cross.



at this point particle would have 2 different sets of initial cond

Digression for conservative systems

$$F = -\nabla V = -\frac{dV}{dx} \quad \text{in 1D}$$

So
$$\boxed{\begin{aligned} \frac{dp}{dt} &= -\frac{dV}{dx} \\ \frac{dx}{dt} &= p/m \end{aligned}}$$

For a particle $K.E. = \frac{p^2}{2m}$

so $\quad \frac{p}{m} = \frac{d}{dp}\left[KE\right]$

Assuming $\quad V = V(x)$

$KE = KE(p)$ then

Total energy looks like

$$H = V(x) + KE(p) \qquad \text{so}$$

H a.k.a. the Hamiltonian

$$\boxed{\begin{aligned} \frac{dp}{dt} &= -\frac{dV}{dx} \\ \frac{dx}{dt} &= p/m \end{aligned}} \qquad \begin{aligned} &= -\frac{\partial H}{\partial x} \\ &= \frac{\partial H}{\partial p} \end{aligned}$$

$$\boxed{\begin{aligned} P_{,t} &= -H_{,x} \\ X_{,t} &= H_{,p} \end{aligned}}$$

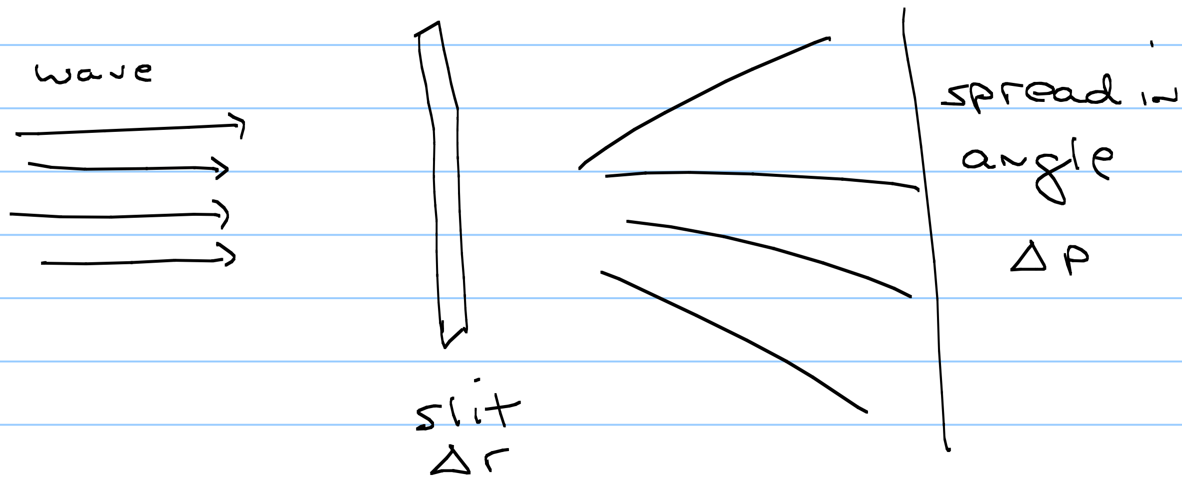
Let $\quad z = \begin{bmatrix} x \\ p \end{bmatrix}$

$$\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = \dot{z} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}_{J} \begin{bmatrix} H_{,x} \\ H_{,p} \end{bmatrix}$$

$$\boxed{\dot{z} = \mp \nabla_z H}$$

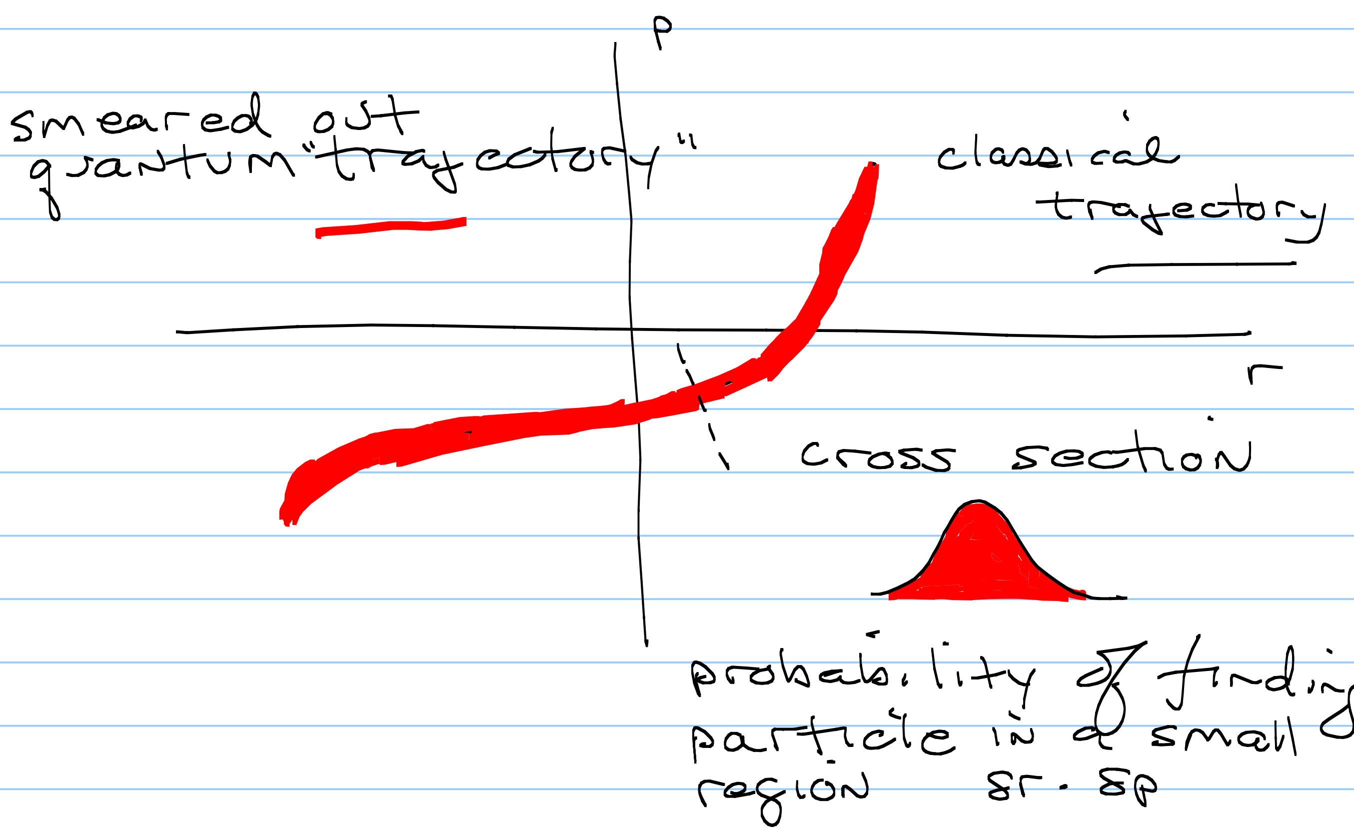
Now, in QM we cannot specify both p and $\bar{r}$ with complete accuracy. If we make $\bar{r}$ more + more precise, then p becomes less and less well known:

wave diffraction analogy



the smaller Δr the bigger Δp

$$\Rightarrow \quad \Delta r \, \Delta p \geq \text{some const.}$$



This probability is given by the following expression

$$|\psi(\vec{r},t)|^2$$

modulus of complex function

$|\psi|^2$ is a probability

ψ is a probability amplitude

ψ is called the wave function and satisfies

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

Schrödinger Equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

must mean that

$$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V$$

Q.M. kinetic energy of particle

Probability Review

coin flip: 2 possible outcomes

die toss: 6 possible outcomes

6-digit lottery: 10^6 possible outcomes

The set of possible outcomes is called the ~~sample description~~ sample description space. Call this Ω

for coin toss $N(\Omega) = 2$

die $N(\Omega) = 6$

lottery $N(\Omega) = 10^6$

Probability of a random event is # of ways of getting that event divided by $N(\Omega)$

$$P[\text{rolling a } 5] = \frac{N(5)}{N(\Omega)} = \frac{1}{6}$$

$$P[\text{rolling} < 5] = \frac{4}{6}$$

So if j is an event in Ω (e.g. heads or tails), then

$$\boxed{P[j] = \frac{N(j)}{N(\Omega)}}$$

1.5 in book

sum over all poss. events

$$\sum_{j}^{\Omega} P(j) = \frac{\sum_j N(j)}{N(\Omega)} = \frac{N(\Omega)}{N(\Omega)} = \underline{1}$$

Example: Box with 5 coins in it
2 pennies, 2 nickles, 1 dime

Select a coin at random
There are 5 possible coins I could select

$$P[\text{penny}] = \frac{2}{5}$$
$$P[\text{nickle}] = \frac{2}{5}$$
$$P[\text{dime}] = \frac{1}{5}$$

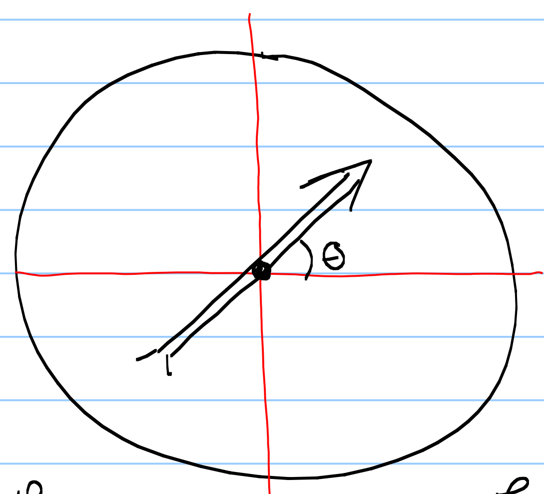
$$\frac{2}{5} + \frac{2}{5} + \frac{1}{5} = 1$$

Numerically valued random events have numbers as outcomes

Parzen's Theory of Probability

> A random (or chance) phenomenon is an empirical phenomenon characterized by the property that its observation under a given set of circumstances does not always lead to the same observed outcomes (so that there is no deterministic regularity) but rather to different outcomes in such a way that there is statistical regularity. By this is meant that numbers exist between 0 and 1 that represent the relative frequency with which the different possible outcomes may be observed in a series of observations of independent occurrences of the phenomenon. ... A random event is one whose relative frequency of occurrence, in a very long sequence of observations of randomly selected situations in which the event may occur, approaches a stable limit value as the number of observations is increased to infinity; the limit value of the relative frequency is called the probability of the random event.

The coin, die, lotto examples are "discrete" phenomena; the outcomes can be mapped into the integers.



spin the arrow. The

outcome is measured by the angle θ.

θ can be any (real) number between $[0, 2\pi]$

what's the probability that after spinning, θ will be exactly, say, π? 0.

$$P\left[\pi/2 \le \theta \le \pi \right] = \frac{\pi/2}{2\pi} = \frac{1}{4}$$

$$P(\theta)\, d\theta = \frac{d\theta}{2\pi}$$

$$\text{mean}\left[\theta\right] = \int_0^{2\pi} \theta \, P(\theta)\, d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \theta \, d\theta = \pi \equiv \overline{\theta}$$

we also call this the Expectation of θ W.R.T. $P(\theta)$

$$\text{variance} \equiv E\left[(\theta - \overline{\theta})^2\right]$$

$$= E(\theta^2) - \overline{\theta}^2$$

Book 1.12

for the spinner

$$\text{variance}(\theta) = \frac{\pi^2}{3}$$

1D For now

$p(x)$ is a prob. density function $\iff$

1) $p(x) \geq 0 \quad \forall x$ Positive
2) $\int_{-\infty}^{\infty} p(x)\, dx = 1$ Normalized

in this course we will write the expectation with $\langle \ \rangle$

E.g.

$$\bar{x} = E[x] = \langle x \rangle = \int_{-\infty}^{\infty} x p(x)\, dx$$

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x) p(x)\, dx$$

$$\text{variance} \equiv \sigma^2 = \langle (x - \bar{x})^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$