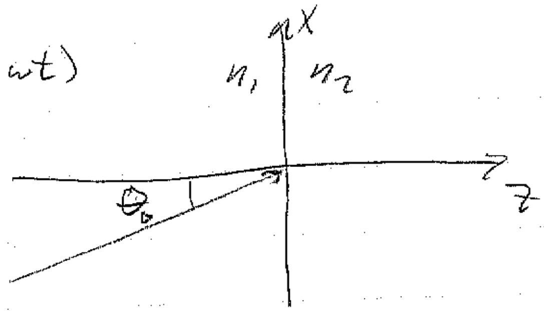
Snell's Law and the Fresnel equations

wave propagating at an angle:

$$E \sim \vec{E}_0 \, e^{i(k_x x + k_z z - \omega t)}$$



with $k_x = k_0 n_1 \sin\theta_0$

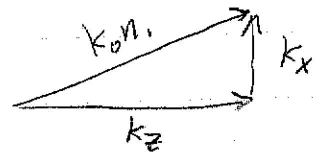
$k_z = k_0 n_1 \cos\theta_0$

from wave equation:

$$\partial_x^2 E + \partial_z^2 E - \frac{\epsilon\mu}{c^2} \partial_t^2 E = 0$$

$$\rightarrow -k_x^2 - k_z^2 + \epsilon\mu \frac{\omega^2}{c^2} = 0$$

$$\therefore \quad k_0^2 n^2 = k_x^2 + k_z^2$$



this vector sum relationship holds true even if $k_x^2 > k_0^2 n_1^2$!

reflection + refraction

suppose $\vec{E}_0 = E_0 \hat{y}$ ("s" polarization, $E \perp$ to POI)

apply continuity at boundary $z = 0$

$$E_0 e^{i k_x^{(0)} x} + E_1 e^{i k_x^{(1)} x} = E_2 e^{i k_x^{(2)} x}$$

this must be true for all x

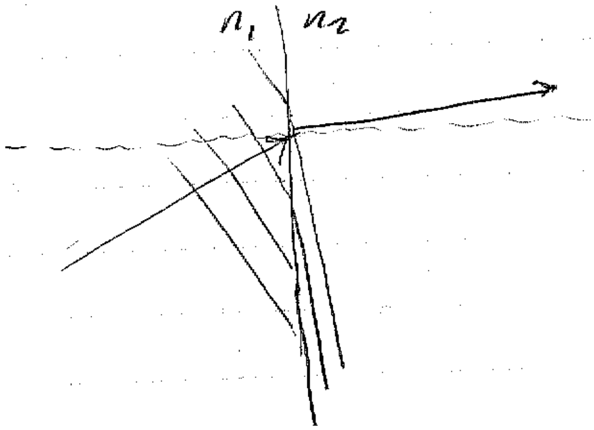
$$\therefore \quad \underset{\text{inc.}}{k_x^{(0)}} = \underset{\text{refl}}{k_x^{(1)}} = \underset{\text{transm.}}{k_x^{(2)}}$$

$$k_0 n_1 \sin\theta_0 = k_0 n_1 \sin\theta_1 = k_0 n_2 \sin\theta_2$$

$\therefore \quad \theta_0 = \theta_1$ $\underline{\theta_{inc} = \theta_{refl}}$

and $\quad n_1 \sin\theta_0 = n_2 \sin\theta_2$ $\underline{\text{Snell's law}}$

Snell's law is about continuity of phase



wave fronts line up

$n_1 < n_2$

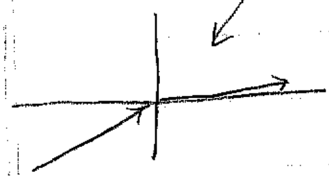
shorter λ inside region 2

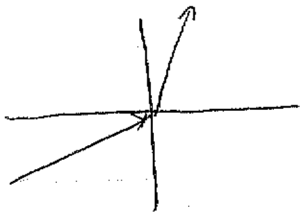
- in a multilayer system $\quad k_0 n_i \sin\theta_i = \underline{\text{constant}}$.

- small angles $\quad n_{inc}\,\theta_{inc} = n_t\,\theta_t \rightarrow \theta_t = \dfrac{n_{inc}}{n_t}\,\theta_{inc}$

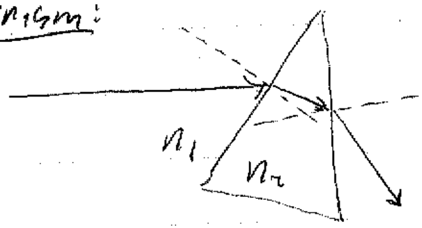
if $n_t > n_{inc}$ refr. toward normal θ_t smaller

$n_t < n_{inc}$ refr. away from normal





prism:

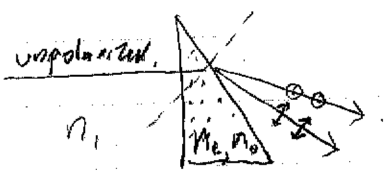


draw surface normal at interface

$n_1 < n_2$

- measure $n(\lambda)$ by measuring θ_{defl}.

birefringent prism



if $n_e < n_o$

$$\theta_{te} = \frac{n_e}{n_1}\,\theta_{inc} < \theta_{to} = \frac{n_o}{n_1}\,\theta_{inc}$$

Alternative views:

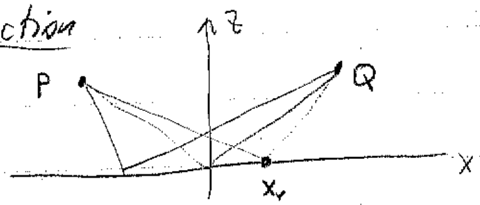
variational method – Fermat's principle of "least time"

optical path:

$$L = \int_P^Q n \, ds$$

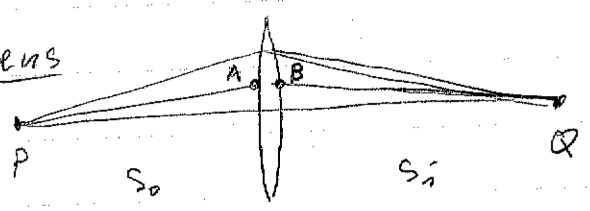
many possible paths, but path taken (classical) or most probable (quantum) is an extremum of L

reflection



vary ray point x_r
minimize total path.

lens

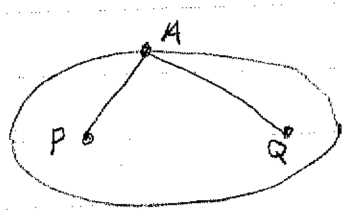


if P, Q are image points, all paths are same length.

- on axis $\overline{PA}$, $\overline{BQ}$ shortest, but $n\,\overline{AB}$ longest.

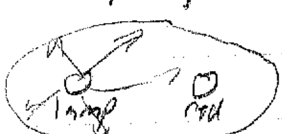
$$\rightarrow \frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

ellipse



path $\overline{PAQ}$ is equal for A on surface of ellipse

used in laser pump chambers:



this approach → foundation for Hamilton's principle of least action and Feynman path integrals.

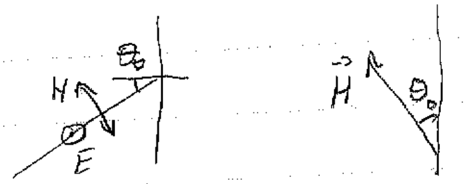
ref QED by Feynman

Fresnel equations: discussion.

see notebook fresnel equations.nb online.

- derivation:
  - matching tangential components of fields → cos() factors.

    on one field



$\to H_0 \cos\theta_0$

- power transmission has angular factors.

$$R = |r|^2 \qquad T = |t|^2 \frac{n_2 \cos\theta_2}{n_1 \cos\theta_0}$$

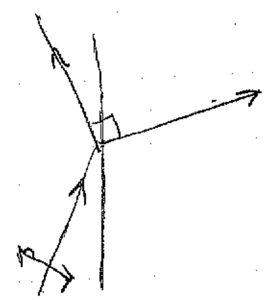
- 'S' and 'P' show different reflectivity:

  Brewster angle for 'P'

  full transmission at $r \to 0$

  at $\theta_1 + \theta_2 = \pi/2$

$$\to \boxed{\tan\theta_B = \frac{n_2}{n_1}}$$



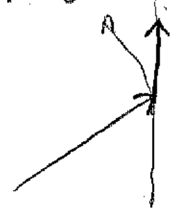
- r, t are in general complex

  for n_1, n_2 real r, t are real unless there is total reflection.

  can use Fresnel eqns for n complex.

TIR

for $n_1 \sin\theta_0 = n_2$ $\quad \theta_2 = \pi/2$



$\sin\theta_{cr} \equiv n_2/n_1$ critical angle for total reflection.

$R = |r|^2 \to 1$

but r is complex for $\theta_0 > \theta_{cr}$.

$r = e^{i\phi}$

phase shift is diff't for 'S', 'P' polariz.