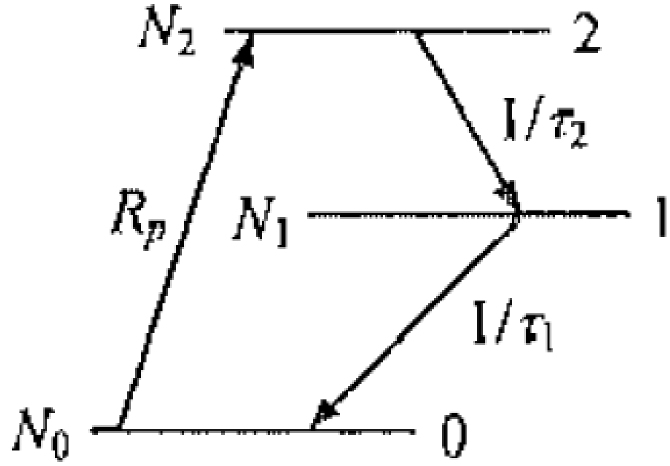
PHGN 480: Laser Physics due 21Sept 2010
Homework 3

1) Consider the energy level scheme shown in the figure below. Starting at t = 0, where the atoms are in the ground state, the atoms are raised from level 0 to level 2 at a pump rate R_p. The lifetime of levels 1 and 2 are τ_1 and τ_2 respectively. Assuming that the ground state 0 is not depleted to any significant extent and neglecting stimulated emission:
   a. Write the rate equations for the population densities, N_1 and N_2, of level 1 and 2 respectively;
   b. Analytically calculate N_1 and N_2 as a function of time. If you want to check your work, you might use DSolve[ ] in Mathematica, but show the steps how to get the solution.
   c. Use Mathematica to plot the population densities (N_1(t) and N_2(t)) and the population inversion (N_2(t) - N_1(t)) on the same plot. Make two plots using the following two input values (you may pick an arbitrary value of R_p):
      i. $\tau_1 = 2$ μs, $\tau_2 = 1$ μs
      ii. $\tau_1 = 1$ μs, $\tau_2 = 2$ μs

   Hint: the differential equation for the population of level 1 can be solved by multiplying both sides by the factor exp(t/τ_1). In this way the left-hand side of the preceding differential equation becomes a perfect differential.



2) Find a relation between spontaneous emission lifetime and cross section for a simple atomic transition that is independent of the dipole moment (take the expressions for each, and eliminate the dipole moment).

3) Svelto problem 2.5.

4) Griffiths problem 9.14. For part (b) see how far you can get without actually doing any integrals: first use the selection rules as described in section 9.3.3 to determine which matrix elements are zero; then, as part of calculating the ratios

see how much cancels before doing the integration. In the end (for part c), you'll have to do the integration, and when you do, you can use Mathematica. Note the spherical harmonics are built-in. As part (d), calculate the natural linewidth for the transition out of the |300> state, and use your relation from problem 2 above to calculate the stimulated emission and absorption cross-sections.

5) Calculate the natural lineshape function assuming the electric field of a decaying state follows the function $E(t) = E_0 \exp\left[-t/2\tau_{sp}\right]\cos\left[\omega_0 t\right]$. Do this using the Fourier transform as we did in class. Make use of the shift theorem.

6) Calculate the Doppler broadened linewidth for
   a. the 488nm transition of an argon ion laser, given that the temperature of the discharge is around 400K
   b. the 632.8nm transition of the HeNe laser, with a discharge temperature of 400K

7) Svelto problem 2.12