MATH 225 – Worksheet #3 Name:________________________________

Sec:________

To get full credit, you must show all of your work.

1. Develop a spreadsheet to solve the following problems using Euler's formula. Make sure to have the headers: k, t, y, and f(t,y) written into your printout. Attach your printouts to this worksheet.
    a. Approximate the IVP: $\frac{dy}{dt} = e^{2t} + y - t$, $y(0) = 0$ with step size of .2 on the interval $0 \le t \le 2$.
    b. Now approximate the same IVP on the same interval with a step size of .1 instead.
    c. The exact solution to this IVP is: $y(t) = e^{2t} + t + 1 - 2e^{t}$. At the endpoint, $t = 2$, which is more accurate part (a) or part (b)?

2. Given $\frac{dy}{dt} = y(y+3)(y-5)$,
    a. Sketch the phase line and classify all of the equilibrium points.

    b. Next to your phase line, sketch the graph of solutions satisfying the initial conditions: $y(0) = -4, y(0) = -1, y(0) = 2, y(0) = 7$. Put your part (b) on one pair of axis.
    c. Describe the long-term behavior, for all of t, of the solution that satisfies the initial condition y(0) = 2.

3. Given $\frac{dy}{dt} = \sin(y^2)$,

   a. Sketch the phase line and classify all of the equilibrium points on the interval $[-\pi, \pi]$.

   b. Next to your phase line, sketch the graph of solutions satisfying the initial conditions: $y(0) = \sqrt{\frac{\pi}{2}}, y(0) = \sqrt{\frac{3\pi}{2}}$. Put your part (b) on one pair of axis.

   c. Describe the long-term behavior, for all of t, of the solution that satisfies the initial condition $y(0) = \sqrt{\frac{\pi}{2}}$.

   d. What theorem guarantees that the solutions with different initial conditions will not cross in the graph from part (b)?

4. Find the solution to the following differential equations or initial-value problems using the Method of Undetermined Coefficients.

   a. $\dfrac{dy}{dt} = 2y + 5e^{-t}$

   b. $\dfrac{dy}{dt} + 2y = 5e^{-2t}$

   c. $\dfrac{dy}{dt} + y = 3\sin(2t), y(0) = 1$