

LINEAR EQUATIONS - COMPLEX/REPEATED/ZERO EIGENVALUES - TRACE DETERMINANT PLANE

1. Given that $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$ where $\mathbf{A} = \begin{bmatrix} -2 & -3 \\ 3 & -2 \end{bmatrix}$.

- (a) Find the real valued general solution of this system.
- (b) Using HPGSYSTEMSOLVER plot the phase portrait and classify the equilibrium solution.

2. Given that $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$ where $\mathbf{A} = \begin{bmatrix} -3 & 10 \\ -1 & 3 \end{bmatrix}$.

- (a) Find the real valued general solution of this system.
- (b) Using HPGSYSTEMSOLVER plot the phase portrait and classify the equilibrium solution.

3. Given that $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$ where $\mathbf{A} = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix}$.

- (a) Find the general solution of this system.
- (b) Using HPGSYSTEMSOLVER plot the phase portrait and classify the equilibrium solutions.

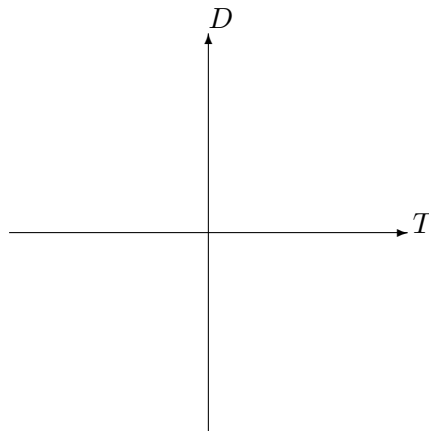
4. Given that $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$ where $\mathbf{A} = \begin{bmatrix} -5 & 1 \\ -1 & -3 \end{bmatrix}$.

- (a) Find the solution of this system assuming that $\mathbf{Y} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$.
- (b) Using HPGSYSTEMSOLVER plot the phase portrait and classify the equilibrium solution.

5. Given the general two-dimensional autonomous system of linear ODE's,

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \tag{1}$$

draw and label all off-axis equilibrium points in the trace-determinant plane.



Using the program TDANIMATION discuss the changes to both the number and classification of equilibrium points as the TD-plane is traversed by the program.

HW6 Soln. (Abbreviated)

1. $A = \begin{bmatrix} -2 & -3 \\ 3 & -2 \end{bmatrix}$

$$\det(A - \lambda I) = (-2 - \lambda)^2 + 9 = \lambda^2 + 4\lambda + 4 + 9 =$$

$$= \lambda^2 + 4\lambda + 13 = 0$$

$$\Rightarrow \lambda = -2 \pm 3i$$

$$[A - \lambda I]\vec{v} = \vec{0} \Rightarrow -3i v_1 = 3v_2 \Rightarrow \vec{v}^{(1)} = \begin{bmatrix} 1 \\ -i \end{bmatrix}, \vec{v}^{(2)} = \begin{bmatrix} 1 \\ i \end{bmatrix}$$

Real Valued Soln.

$$\vec{Y}(t) = k_1 e^{-2t} \begin{bmatrix} \cos(3t) \\ \sin(3t) \end{bmatrix} + k_2 e^{-2t} \begin{bmatrix} \sin(3t) \\ -\cos(3t) \end{bmatrix}$$

2) $\det \begin{bmatrix} -3-\lambda & 10 \\ -1 & 3-\lambda \end{bmatrix} = (3-\lambda)(-3-\lambda) + 10 =$

$$= \lambda^2 - 9 + 10 = 0$$

$$\Rightarrow \lambda = \pm i$$

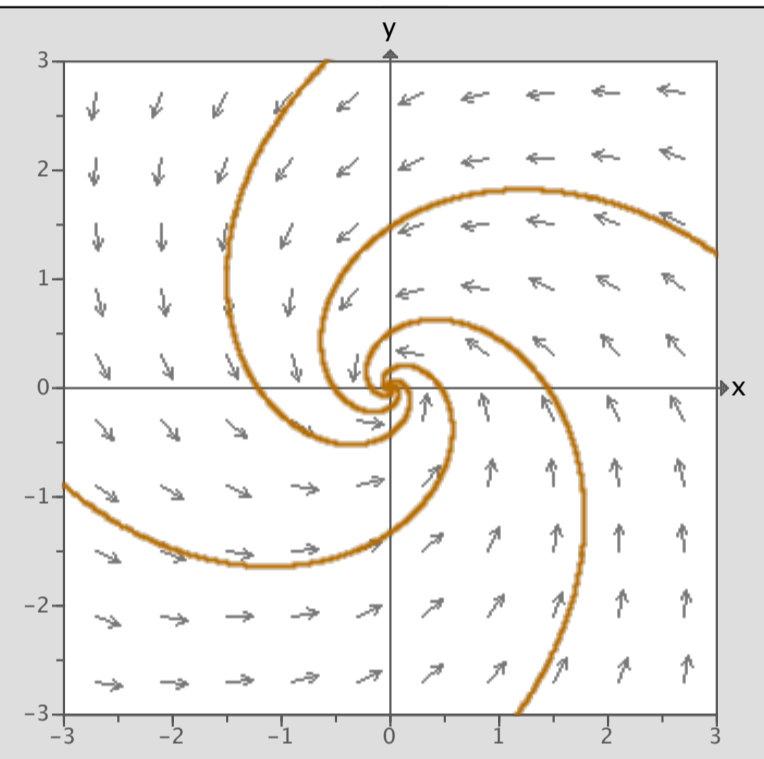
$$[A - \lambda I]\vec{v} = \vec{0} \Rightarrow (-3 + -i)v_1 + 10v_2 = 0 \Rightarrow v_1 = 10, v_2 = 3+i$$

$$\Rightarrow \vec{v}^{(1)} = \begin{bmatrix} 10 \\ 3+i \end{bmatrix}, \vec{v}^{(2)} = \begin{bmatrix} 10 \\ 3-i \end{bmatrix}$$

Real Valued Soln

$$\vec{Y}(t) = k_1 \begin{bmatrix} 10 \cos(3t) \\ 3 \cos(3t) - \sin(3t) \end{bmatrix} + k_2 \begin{bmatrix} 10 \sin(3t) \\ 3 \sin(3t) + \cos(3t) \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



clear eigenlines

spiral sink

x(0) =

y(0) =

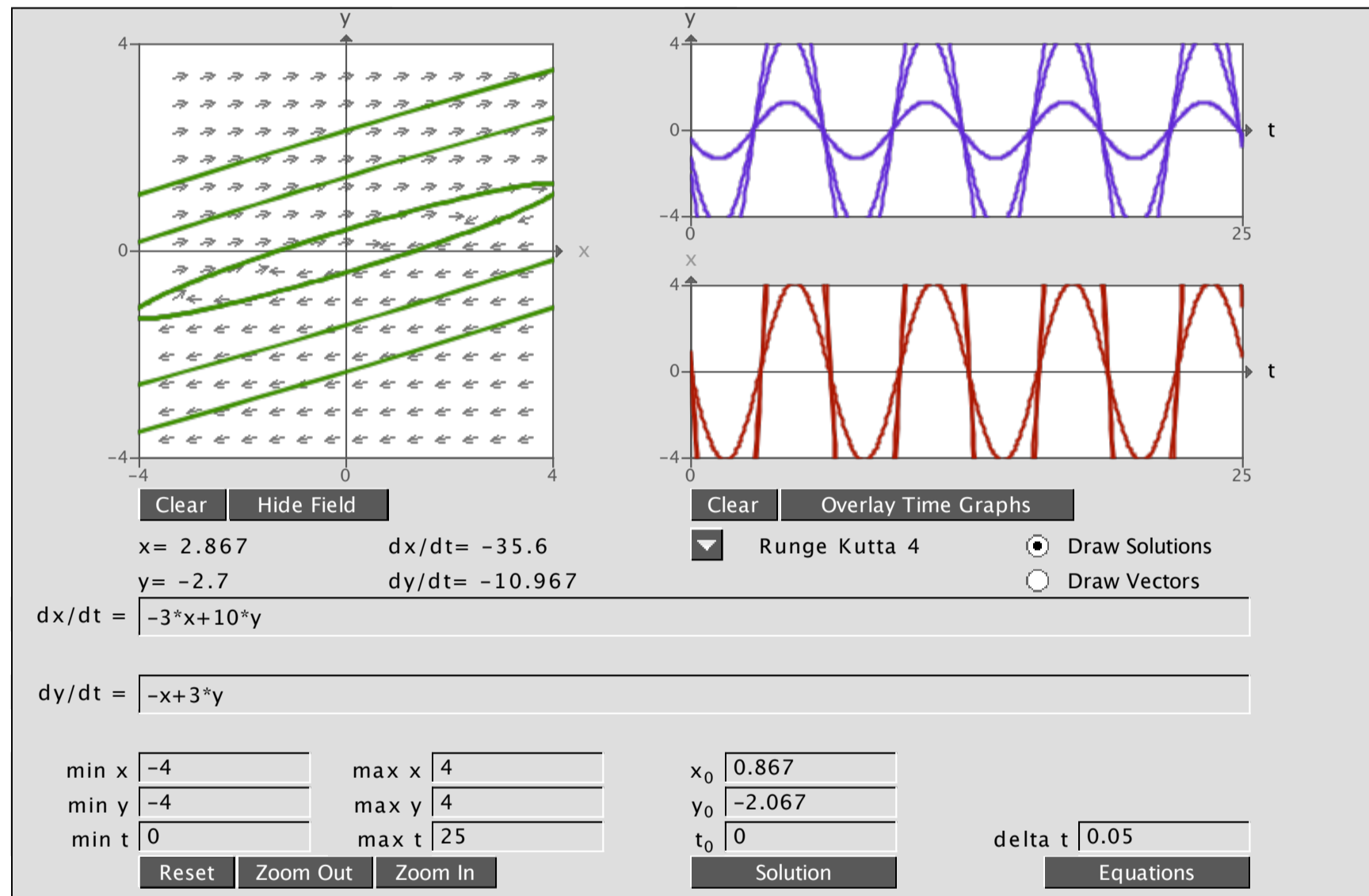
a -2.00

b -3.00

c 3.00

d -2.00

trace & determinant eigenvalues



$$3) \textcircled{a} \det(A - \lambda I) = \det \begin{pmatrix} -3-\lambda & 1 \\ 1 & -1-\lambda \end{pmatrix} =$$

$$= (-3-\lambda)(-1-\lambda) - 1 = \lambda^2 + 3\lambda + \lambda + 3 - 1 =$$

$$= \lambda^2 + 4\lambda = \lambda(\lambda + 4) = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = -4$$

$$[A - \lambda_1 I] \vec{v} = \vec{0} \Rightarrow 3v_1 = v_2 \Rightarrow \vec{v}^{(1)} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$[A - \lambda_2 I] \vec{v} = \vec{0} \Rightarrow \vec{v}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

General Soln:

$$\vec{Y}(t) = k_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{0t} + k_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-4t}$$

$$4) \textcircled{b} \det(A - \lambda I) = (-5-\lambda)(-3-\lambda) + 1 =$$

$$= \lambda^2 + 8\lambda + 15 + 1 = \lambda^2 + 8\lambda + 16 = (\lambda + 4)^2 = 0$$

$$\lambda = -4$$

$$[A - \lambda I] \vec{v} = \vec{0} \Rightarrow \vec{v}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

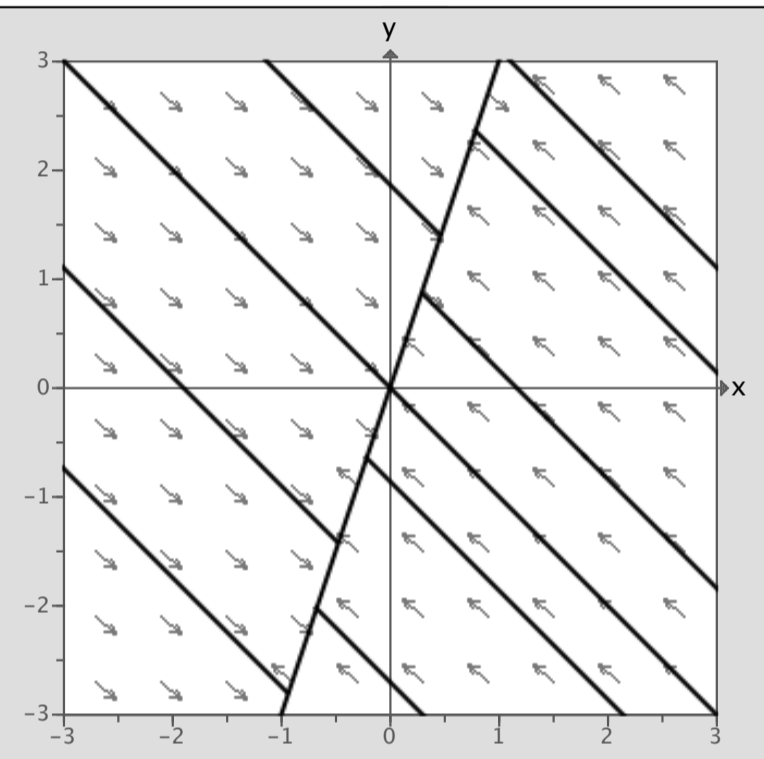
General Soln:

$$\vec{Y}(t) = \left(\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) e^{-4t}$$

5) See Text or Class notes for T-D diagram.
Also see T-D animation.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix}$$

trace & determinant eigenvalues



clear eigenlines

zero eigenvalue

x(0) =

y(0) =

a -3.00

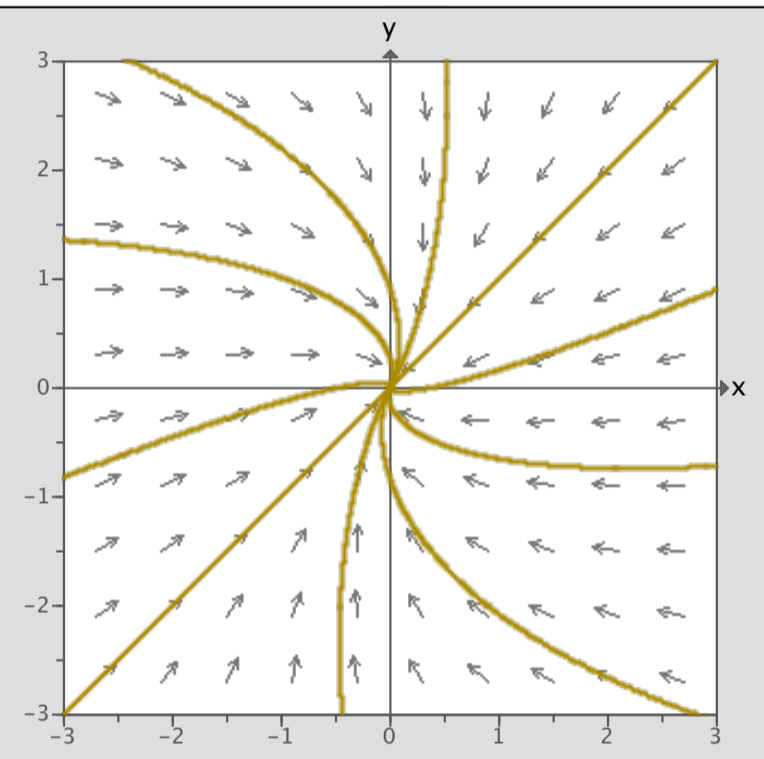
b 1.00

c 3.00

d -1.00

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

trace & determinant eigenvalues



clear eigenlines
repeated eigenvalues

x(0) =
y(0) =

