where c is a constant, $c = (dq/d\rho)(\rho_0)$. The initial condition is that the initial *perturbed* traffic is known,

$$\rho_1(x, 0) = f(x).$$

c has the dimensions of a velocity (why?). It will be shown to be a very important velocity.

Although there seems no motivation to do so, let us introduce a new spatial coordinate x' moving with the constant velocity c. Let us assume that the two spatial coordinate systems x and x' have the same origin at $t = 0$: See Fig. 67-1. At a later time t, the moving coordinate system has moved a

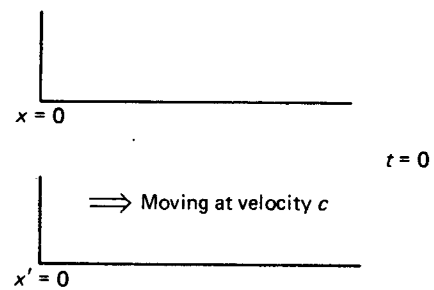

Figure 67-1.

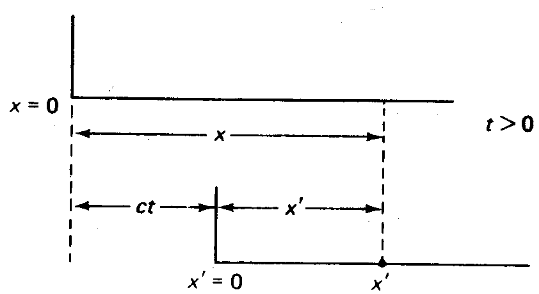

Figure 67-2 Frame of reference x' moving at velocity c.

distance ct (since it is moving at a constant velocity c) as shown in Fig. 67-2. Thus, if $x' = 0$, then $x = ct$. Furthermore at x', $x = ct + x'$ or

$$\boxed{x' = x - ct.}$$

We will first investigate what happens to the partial differential equation describing linearized traffic flow in this moving coordinate system. Instead of the solution depending on x and t, it will depend on x' and t. However, in making changes of variables involving partial derivatives, it is more convenient

to introduce different notations for each set of variables. Thus for the moving coordinate system, we use the variables x' and t', where $t' = t$. Consequently, the change of variables we use is

$$\boxed{\begin{aligned} x' &= x - ct \\ t' &= t. \end{aligned}}$$

In order to express the partial differential equation in terms of the new variables, the chain rule of partial derivatives is used:

$$\frac{\partial}{\partial x} = \frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} + \frac{\partial t'}{\partial x}\frac{\partial}{\partial t'}$$

$$\frac{\partial}{\partial t} = \frac{\partial x'}{\partial t}\frac{\partial}{\partial x'} + \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'},$$

and thus

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'}$$

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}.$$

Note that even though $t' = t$, $(\partial/\partial t') \neq (\partial/\partial t)$. The reason for this is clear from the definitions of these two partial derivatives. $\partial/\partial t$ means the time derivative keeping x fixed (that is, in a stationary coordinate system), while $\partial/\partial t'$ means the time derivative keeping x' fixed (that is, in a coordinate system moving with velocity c). The changes in time may be different in the two systems. This emphasizes the importance of introducing a new time variable t'; it enables us to have a notational distinction between keeping x fixed and keeping x' fixed.

In this manner, this partial differential equation in a coordinate system moving with velocity c becomes

$$-c\frac{\partial \rho_1}{\partial x'} + \frac{\partial \rho_1}{\partial t'} + c\frac{\partial \rho_1}{\partial x'} = 0,$$

which simplifies and becomes

$$\frac{\partial \rho_1}{\partial t'} = 0.$$

This partial differential equation can be directly integrated (see Sec. 65). If x' is fixed, ρ_1 is constant; that is ρ_1 is constant in time in a coordinate system moving with velocity c. For different values of x', ρ_1 may be a different