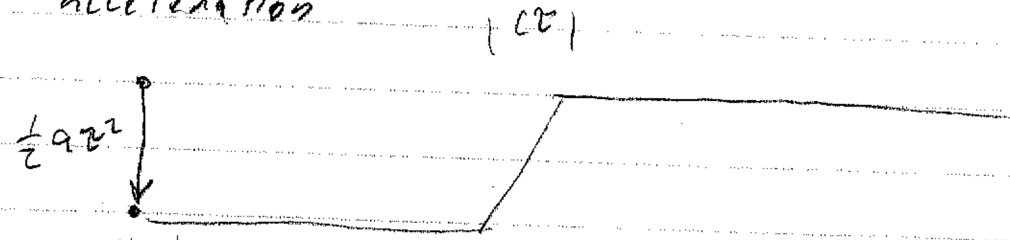
Qualitative analysis of radiation from impulsive acceleration

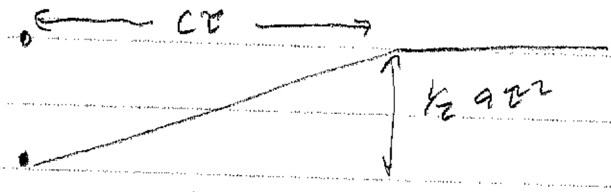


initial velocity not important
Observe at large distance
kink in $\vec{E}$ propagates out at c

final velocity $u = a\tau$

compare $\frac{1}{2}a\tau^2$ to $c\tau$ $\rightarrow \frac{1}{2}a\tau < c$

trace kink back to source:

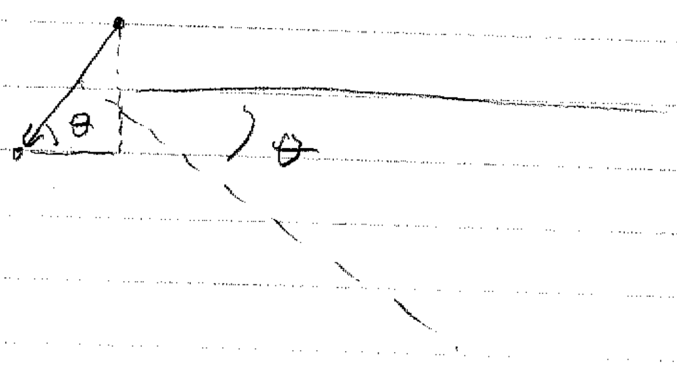


$$\frac{E_\perp}{E_\parallel} \sim \frac{\frac{1}{2}a\tau^2}{c\tau} = \frac{a\tau}{2c}$$

$E_\parallel$ at start of accel $\sim q/r^2$ w/ $r \sim c\tau$

$E_\perp \sim \frac{a\tau}{2c} \cdot \frac{q}{r^2}$ elim. τ $\rightarrow \frac{qa}{2c^2} \cdot \frac{1}{r}$

if $\vec{a}$ is not $\perp$ to $\vec{r}$



$\rightarrow \sin\theta$ factor

Radiation w/ velocity ∥ acceleration

$$\vec{E}_a = \frac{e}{c^2 K^3 R^3}\,\vec{R}\left(\left(\vec{R}-\cancel{\vec{\beta}R}\right)\times\vec{a}\right)$$

0 since here $\vec{\beta}\times\vec{a}=0$

now $\vec{S}_a = \vec{S}_a(\beta=0)\cdot K^{-6}$

but this is power radiated from charge:

$$\frac{d\mathcal{E}_{tot}}{dt} = -\int \vec{S}_a\cdot d\vec{A}$$

we want radiated power measured at distant point.

$$P = -\frac{d\mathcal{E}_{tot}}{dt'} = -\frac{d\mathcal{E}}{dt}\frac{dt}{dt'} \qquad t' = t - R(t')/c$$

↳ retarded time

$$\frac{dt}{dt'} = 1 + \frac{1}{c}\frac{d}{dt'}\left|\vec{r}-\vec{r}_e(t')\right| = 1 + \sum \frac{u_i}{c}\frac{d}{dx_{ei}}\left|\vec{r}-\vec{r}_e\right|$$

$$\left|\vec{r}-\vec{r}_e\right| = \sqrt{r^2 + r_e^2 - 2\vec{r}\cdot\vec{r}_e}$$

$$\frac{d}{dx_{ei}}\left|\vec{r}-\vec{r}_e\right| = \frac{1}{2}\frac{1}{\sqrt{}}\left(2x_{ei}-2x_i\right)$$

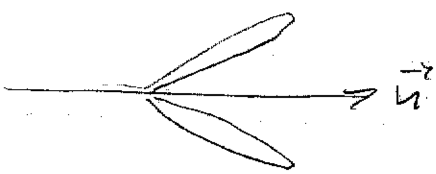
$$\sum \beta_i \frac{d}{dx_{ei}}\left|\vec{r}-\vec{r}_e\right| = \sum \frac{\beta_i\left(x_{ei}-x_i\right)}{R} = -\frac{\vec{\beta}\cdot\vec{R}}{R}$$

$$\frac{dt}{dt'} = 1 - \frac{\vec{\beta}\cdot\vec{R}}{R} = K = 1-\beta\cos\theta$$

↳ measured rel to $\vec{u}$

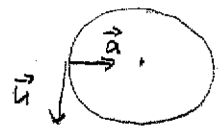
now we have K^{-5} in $dP/d\Omega$

this directs lobes toward $\vec{u}$

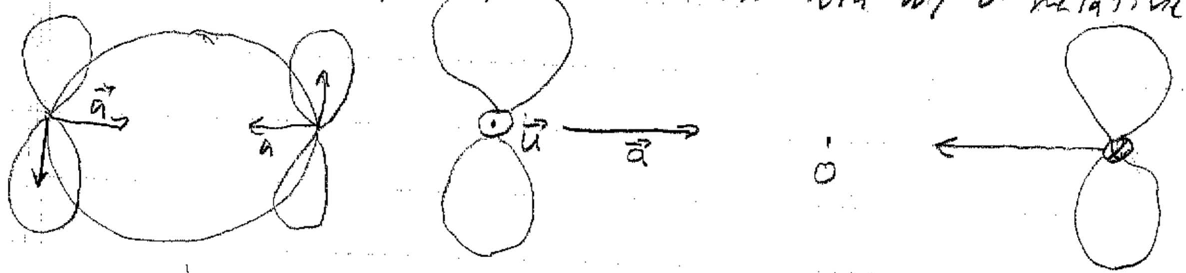


Synchrotron radiation - circular orbit.

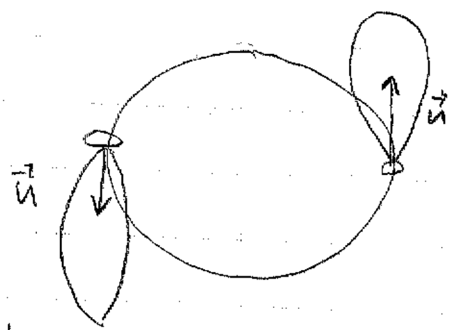
$\vec{u} \perp \vec{a}$



At low speeds, Larmor formula w/ θ relative to $\vec{a}$



higher speeds: push lobes forward, toward $\vec{u}$



"searchlight"

output: pulses w/ rep rate 1/orbit period

Antennas - radiation from collections of charges

2 assumptions:

1) localized charge/current distribution (size d)
2) source oscillates w/ frequency ω ($\lambda = 2\pi c/\omega$)

Observation distance - r

Must make approximations to do calculations: order these dimensions

Scaling analysis:

$$\vec{E} = e\left[\frac{(\hat{R}-\beta)(1-\beta^2)}{K^3R^2} + \frac{\hat{R}\times((\hat{R}-\vec{\beta})\times\vec{a})}{c^2K^3R}\right]$$

$$\vec{B} = \hat{R}\times\vec{E}, \quad \vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{H})$$

compare relative magnitude of terms.

K^3 common to all

single charge: oscillates over dist d at freq. ν =

$u \sim d\cdot\nu \quad \beta \sim d\nu/c \sim d/\lambda \quad a/c^2 \sim d/\lambda^2$

since $\beta < 1$, single charge must have $d < \lambda$

example: $\nu \sim 1$ MHz (AM radio)

$\rightarrow \lambda \sim 300$ m

$d \sim 1$ cm

$u \sim d\nu \sim 10^{-2}\,m \cdot 10^6\,s^{-1} \sim 10^4$ m/s

$\rightarrow \beta \sim d/\lambda \sim 3\times10^{-5}$

100 MHz FM

3 m

10^6 m/s

3×10^{-3}

$$E \sim \frac{1}{R^2} + \frac{a}{c^2R} \sim \frac{1}{R^2} + \frac{d}{\lambda^2 R}$$

$$S \sim \frac{1}{R^4} + \frac{2d}{\lambda^2R^3} + \frac{d^2}{\lambda^4R^2} \sim 1,\ \frac{2dR}{\lambda^2},\ \frac{d^2R^2}{\lambda^4}$$