MATH 225 - Differential Equations May 15, 2008
Homework 3, Field 2008 **Due**: May 20, 2008

QUALITATIVE ANALYSIS - EXISTENCE AND UNIQUENESS - PHASE LINE

1. Section 1.3 of the text, problems 8, 10, 15.

2. Consider the following logistics models for population growth,

$$\frac{dP}{dt} = f_H(P) = kP\left(1 - \frac{P}{N}\right) - H \tag{1}$$

$$\frac{dP}{dt} = f_\alpha(P) = kP\left(1 - \frac{P}{N}\right) - \alpha P \tag{2}$$

where k, N, M, H, α are the growth rate, carrying capacity, minimum threshold, harvesting and harvesting rate parameters respectively.

   (a) For (1) let $k = N = 2$ and $H = 0.5$. Using HPGSOLVER, with the domain $t \in (-3, 5)$ and $y \in (-1, 5)$, to plot the slope field and solutions associate the initial conditions $(0, .25)$, $(0, .5)$ $(0, 3)$ and discuss the long term behavior for each solution.

   (b) For (2) let $k = N = 2$ and $\alpha = 0.5$. Using HPGSOLVER, with the domain $t \in (-3, 5)$ and $y \in (-1, 5)$, to plot the slope field and solutions associate the initial conditions $(0, .125)$, $(0, .25)$ $(0, 5)$ and discuss the long term behavior for each solution.

   (c) Compare these two harvesting models, which would you use to harvest a population where P cannot be exactly known? What if you could always know exactly the population P, which would you use then?

3. Assuming f satisfies the hypotheses of the Uniqueness Theorem and that $y_1(t) = 4+t+3t^2$ and $y_2(t) = \dfrac{1}{t^2 + 2t + 3}$ are solutions to $\dfrac{dy}{dt} = f(t, y)$. What can you conclude about the solution to $\dfrac{dy}{dt} = f(t, y)$ where $y(0) = \dfrac{1}{2}$ for all $t \in \mathbb{R}$?

4. Given $\dfrac{dy}{dt} = y(y - 2)(y - 4)$,

   (a) Sketch the phase line and classify all equilibrium points.

   (b) Next to your phase line, sketch the solutions satisfying the initial conditions $y(0) = -1$, $y(0) = 1$, $y(0) = 3$, and $y(0) = 5$.

   (c) Describe the long-term behavior of the solution that satisfies the initial condition $y(0) = 1$.

5. Given $\dfrac{dy}{dt} = \sin(y^2)$,

   (a) Sketch the phase line and classify all equilibrium points. (You cannot sketch the entire phase line in this case, but show at least 5 equilibrium points, including both positive and negative y-values.)

   (b) Next to your phase line, sketch the solutions satisfying the initial conditions $y(0) = \sqrt{\dfrac{\pi}{2}}$, and $y(0) = \sqrt{\dfrac{3\pi}{2}}$.

   (c) Describe the long-term behavior of the solution that satisfies the initial condition $y(0) = \sqrt{\dfrac{\pi}{2}}$.