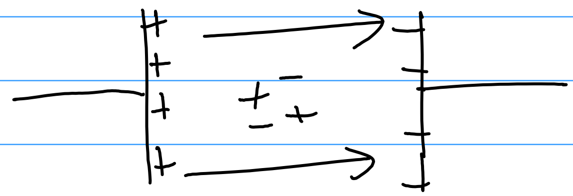
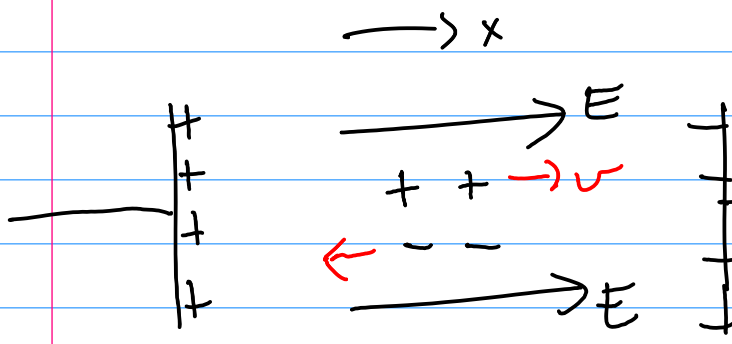
Lecture 11 Shadowitz: Chapter 2 appendix 2 and 4-1

InkSurvey question





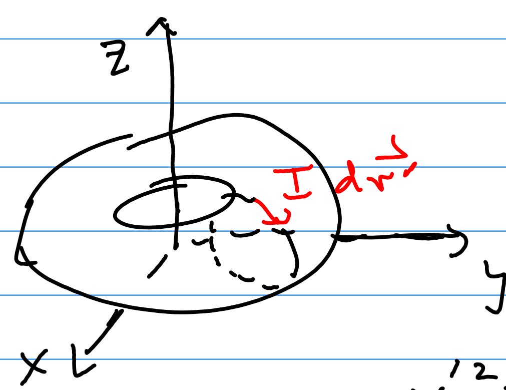
$$I_+ = \rho_0 v$$

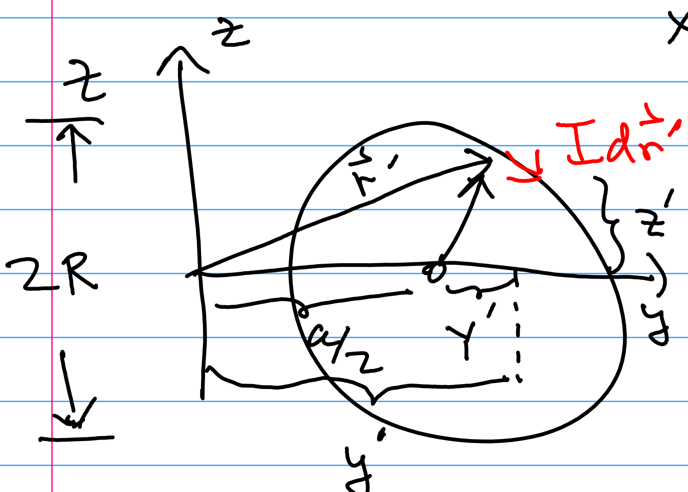
$$I_- = (-\rho_0)(-v)$$

$$\vec F = q\vec E + q\vec v \times \vec B$$

The two currents in the same direction attract each other in this simple model.

InkSurvey question





$$y'^2 + z'^2 = R^2$$

$$\vec r' = y'\hat y + z'\hat z$$

$$\vec r' = \left(\frac{a}{2} + Y'\right)\hat y + \sqrt{R^2 - Y'^2}\,\hat z$$

$$d\vec r' = dY'\hat y + \frac{1}{2}\frac{-2Y'dY'}{\sqrt{\quad}}\hat z$$

This looks like a multivalued function. However sign is needed for the lower half of circle.

$$\vec r' = \left(\frac{a}{2} + Y'\right)\hat y \pm \sqrt{R^2 - Y'^2}\,\hat z$$

Muddiest points:

1.) Other books with examples: Griffiths, Pollack and Stump, Lorain and Coursan Purcell Berkely Physics series, Feynman lectures on E&M

2.) Unanswered questions: Please email me or come and see me where I can give individual attention.

- -derivation of Ampere's law
- -how to find dr in general
- -Ampere's law and $\partial \vec{E}/\partial t$
- -Green's theorem

$$\int (T\nabla^2 u - u\nabla^2 T)\, d\text{volume} = \oint (T\vec{\nabla} u - u\vec{\nabla} T)\cdot d\vec{a}$$

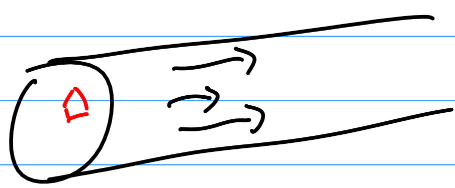
3.) What is the flux for multiple B's

$$\int \vec{\nabla}\times\vec{B}_{tot}\cdot d\vec{a} = \int \vec{\nabla}\times\vec{B}_1\cdot d\vec{a} + \int \vec{\nabla}\times\vec{B}_2\cdot d\vec{a}$$

4.) We use Helmholtz theorem just to know we have a unique solution for E and B.

5.) More practice on trajectories of particles in E and B.

6.) Why draw a surface or tile around a current if curl B is at a point?



$\vec{J} = \alpha r$

J is not just at a point

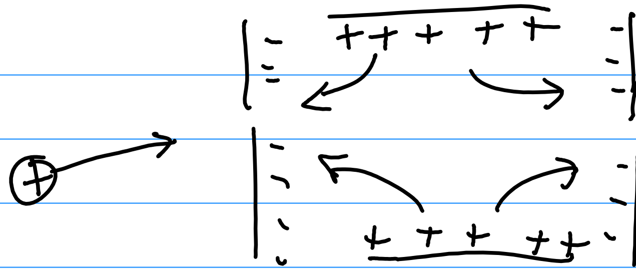
Here Ampere's law is applied over the cross section of the wire.

$$\int \vec{\nabla}\times\vec{B}\cdot d\vec{a} = \oint \vec{B}\cdot d\vec{l}$$

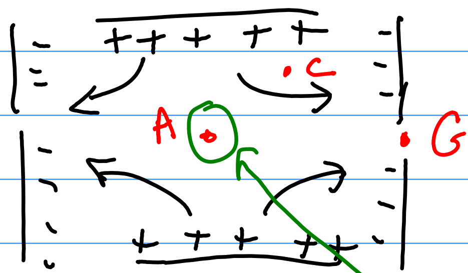
$$\int \mu_0 \vec{J}\cdot r\,dr\,d\varphi\,\hat{z} = \iint \alpha r\,\hat{z}\cdot r\,dr\,d\varphi\,\hat{z}$$

7.) I'll work on the exam this weekend and let you know on Monday the coverage.

Electrostatic lens



What is the div of E at these points? Use divergence theorem to answer.



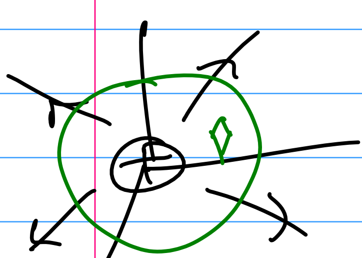
$$\vec\nabla\cdot\vec E = 0$$

$$\int \vec\nabla\cdot\vec E \, d\text{volume} = \oint \vec E\cdot d\vec a = 0$$

No charge enclosed!

Let's apply the divergence theorem to the field from a point charge at the origin.

$$\vec v = \frac{\hat r}{r^2} \qquad \vec\nabla\cdot\vec v = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) = 0, \quad v_r = \frac{1}{r^2}$$



$$\oint \vec E\cdot d\vec a = \int kQ\frac{\hat r}{\cancel{r^2}}\cdot\cancel{r^2}\sin\theta\, d\theta\, d\phi\, \hat r = kQ\,4\pi$$

$$\underbrace{\int \vec\nabla\cdot\vec v \, d\text{volume}}_{=0} \neq \oint \vec v\cdot d\vec a$$

Why is the LHS not equal to the RHS?

Note that we are dividing by zero at one point inside the volume. $\frac{\hat{r}}{r} \to \infty$

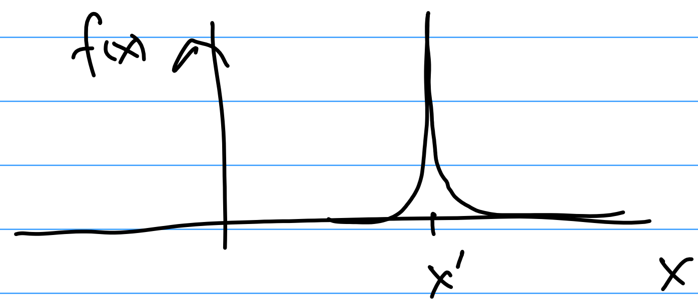
$@\ r = 0$

Divide by zero in algebra: $0 \times 1 = 0 \times 2$

$$\frac{0 \times 1}{0} = \frac{0 \times 2}{0} \Rightarrow 1 = 2$$

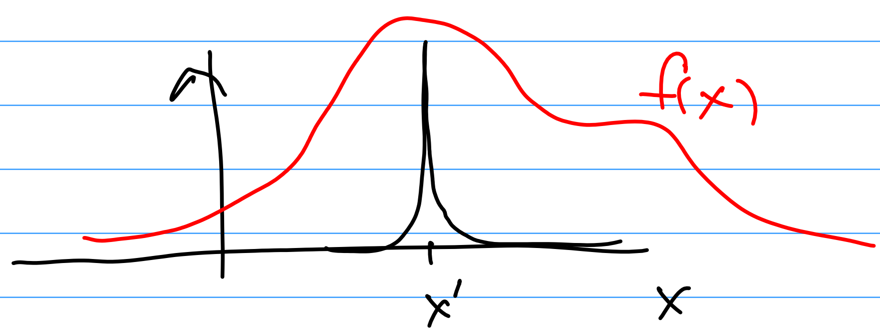
How do we deal with this infinity?

Delta function:



$$f(x) = \delta(x - x')$$

$$\int_{-\infty}^{\infty} \delta(x - x')\, dx = 1$$



$$\int_{-\infty}^{\infty} f(x)\, \delta(x - x')\, dx = f(x')$$

How to you calculate delta functions in 3-D (congruous)?

$$\int \delta^3(\vec{r})\, d\,\text{volume} = \int \delta(x)\delta(y)\delta(z)\, dx\, dy\, dz = 1$$

To satisfy the diverence theorem we must have

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2} = 4\pi \delta^3(\vec{r})$$

$$\vec{\nabla} \cdot \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = 4\pi \delta^3(\vec{\mathscr{r}}) = 4\pi \delta^3(\vec{r} - \vec{r}\,')$$

$$\vec{E} = k \frac{Q}{\mathscr{r}^2} \hat{\mathscr{r}}$$

$$\vec{E} = \int k \frac{\rho(r') \, dx' dy' dz'}{\mathscr{r}^2} \hat{\mathscr{r}}$$

$$\vec{\nabla} \cdot \vec{E} = \int k \rho(r') \vec{\nabla} \cdot \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \, dx' dy' dz'$$

$$= \int k \rho(r') \, 4\pi \delta^3(\vec{r} - \vec{r}\,') \, dx' dy' dz'$$

$$= k \rho(r) 4\pi = \frac{1}{4\pi \epsilon_0} 4\pi \rho(r)$$

$$= \boxed{\frac{\rho(x,y,z)}{\epsilon_0} = \vec{\nabla} \cdot \vec{E}(x,y,z)}$$

How is this similar to our results for B (analogy)?

Both the expressions for div E and curl B have the dependence on x,y,z

1.) We derived curl of B in the following way

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,') \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\text{vol} \;:\; \vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\int \vec{\nabla}\times\left[\frac{\vec{J}(\vec{r}\,')\times\hat{\mathscr{r}}}{\mathscr{r}^2}\right] d\text{vol}$$

Which resulted in $\vec{\nabla}\times\vec{B}(x,y,z) = \mu_0 \vec{J}(x,y,z)$

$$\frac{\rho(x,y,z)}{\epsilon_0} = \vec{\nabla}\cdot\vec{E}(x,y,z)$$

Note the same x,y,z dependence even though both integrals for E and B had the primed variables in them!

Next we need to find the curl of E to uniquely determine E.

First not that the curl operator is linear so the superposition prin. holds.

$$\vec{\nabla}\times\vec{E}_{tot} = \vec{\nabla}\times\vec{E}_1 + \vec{\nabla}\times\vec{E}_2 + \dots$$

$$\vec{E} = \int k \frac{\rho(r')\,dx'dy'dz'}{\mathscr{r}^2}\hat{\mathscr{r}}$$

$$\vec{\nabla}\times\vec{E} = \int k\,\rho(r')\,dx'dy'dz'\;\underbrace{\vec{\nabla}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}}_{0} = \phi$$

from your homework

Statement $\vec{\nabla} \times \vec{\nabla} V(x, y, z) = 0$

Questions: How do you calculate or show this (congruous)? Just write it out in cartesian coords.

$$\vec{\nabla} V \equiv \hat{x}\frac{\partial}{\partial x}V + \hat{y}\frac{\partial}{\partial y}V + \hat{z}\frac{\partial}{\partial z}V$$

$$\vec{\nabla} \times \vec{\nabla} V(x, y, z) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \end{vmatrix}$$

$$= \hat{x}\underbrace{\left(\frac{\partial^2 V}{\partial z \partial y} - \frac{\partial^2 V}{\partial y \partial z}\right)}_{\emptyset} - \hat{y}\underbrace{\left(\frac{\partial^2 V}{\partial x \partial z} - \frac{\partial^2 V}{\partial z \partial x}\right)}_{0} + \underbrace{\hat{z}\left(\quad \right)}_{0}$$

We can therefore write $\vec{E} = -\vec{\nabla} V$

Convention

$$\int \vec{\nabla} \times \vec{E} \cdot d\vec{a} = \oint \vec{E} \cdot d\vec{r}$$

Fundamental theorem of gradients.