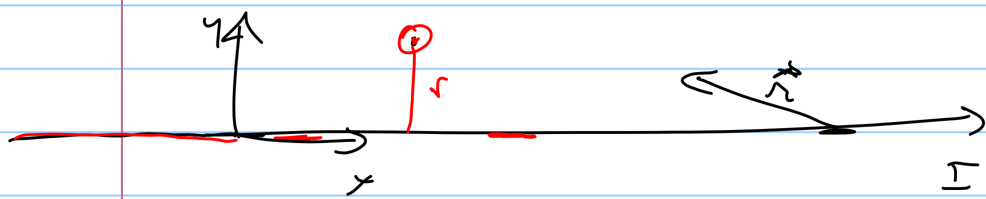
Lecture 28

Note Title 3/31/2006

finding $\vec{B}$ from an ∞ wire



$$\int d\vec{B} = \int \mu_0 I \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$$\oint |\vec{B}|\,|d\vec{l}| \underbrace{\cos\theta}_{1} = |\vec{B}| \underbrace{\oint dl}_{2\pi r} = \mu_0 I_0 \qquad B = \frac{\mu_0 I}{2\pi r}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{A} = \vec{A}' + \vec{\nabla}\psi$$

scalar

$$\vec{B}' = \vec{\nabla} \times \vec{A}' + \underbrace{\vec{\nabla} \times \vec{\nabla}\psi}_{0} = \vec{B}$$

$PE = mgh + \underbrace{mgH}_{\text{const}}$

$\vec{F} = -\vec{\nabla} PE$

To uniquely define $\vec{A}$ need two eqn

$\vec{\nabla} \times \vec{A} = \underline{\vec{B}}$ & $\vec{\nabla} \cdot \vec{A} =$ arbitrary

$\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot \vec{A}' + \vec{\nabla} \cdot \vec{\nabla} \psi$

$= \vec{\nabla} \cdot \vec{A}' + \nabla^2 \psi$ = arbitrary

↑ arbitrary

Magnetostatics $\boxed{\vec{\nabla} \cdot \vec{A} = 0}$ Coulomb gauge

$\vec{\nabla} \cdot \vec{A} = -\frac{1}{c^2} \frac{\partial V}{\partial t}$ Lorentz gauge (EM radiation)

\begin{tikzcd}
 & dq \arrow[dl] \arrow[dr] & \\
dV & & d\vec{E} \arrow[ll]
\end{tikzcd}

$d\vec{E} = k\frac{dq}{r^2}\hat{r}$

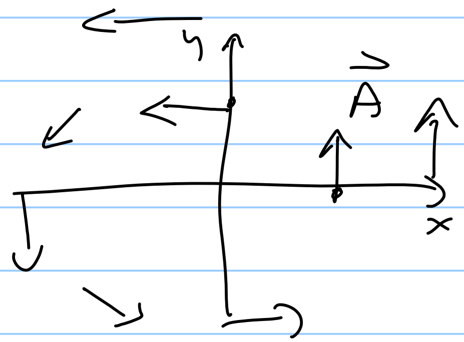
$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{in}}{\epsilon_0}$

\begin{tikzcd}
 & \vec{J} \arrow[dl] \arrow[dr] & \\
\vec{A} \arrow[rr, shift right, "\vec{B}=\vec{\nabla}\times\vec{A}"'] & & \vec{B} \arrow[ll, shift right]
\end{tikzcd}

Biot & Savart

Amps Law

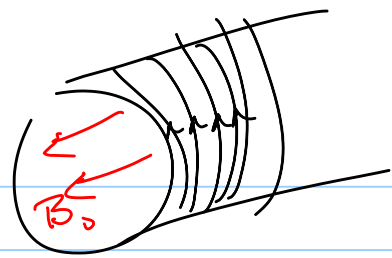
$A_x = -\frac{B_0 y}{2}, \quad A_y = \frac{B_0 x}{2}, \quad A_z = 0$



$$\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{B_0 y}{2} & \frac{B_0 x}{2} & 0 \end{vmatrix}$$

$$\vec{B} = B_0\,\hat{z}$$

Solenoid



$\vec{A}$ & $\vec{J}$ same direction

$\vec{\nabla}\psi$

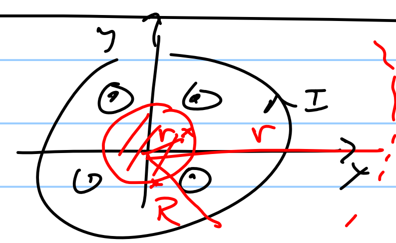
$\vec{A}$ could also be written $\quad A - \vec{\nabla}\left(\frac{B_0 x y}{2}\right)$

$$= -\frac{B_0 y}{2}\hat{x} + \frac{B_0 x}{2}\hat{y} - \frac{B_0 y}{2}\hat{x} - \frac{B_0 x}{2}\hat{y}$$

$$\Rightarrow \quad A_x = -B_0 y \qquad A_y = 0 \qquad A_z = 0$$

$$\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -B_0 y & 0 & 0 \end{vmatrix} = B_0 \hat{z}$$

Solenoid generates field

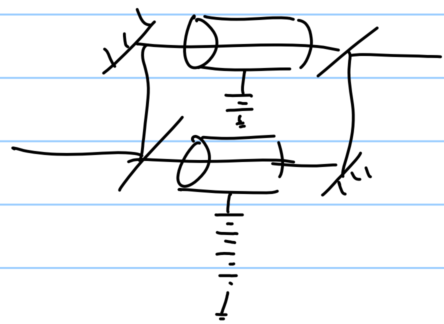


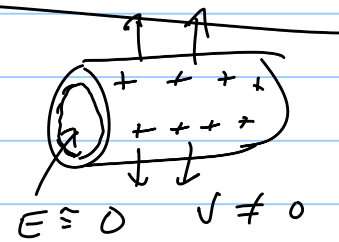
given B we could find $\vec{A}$

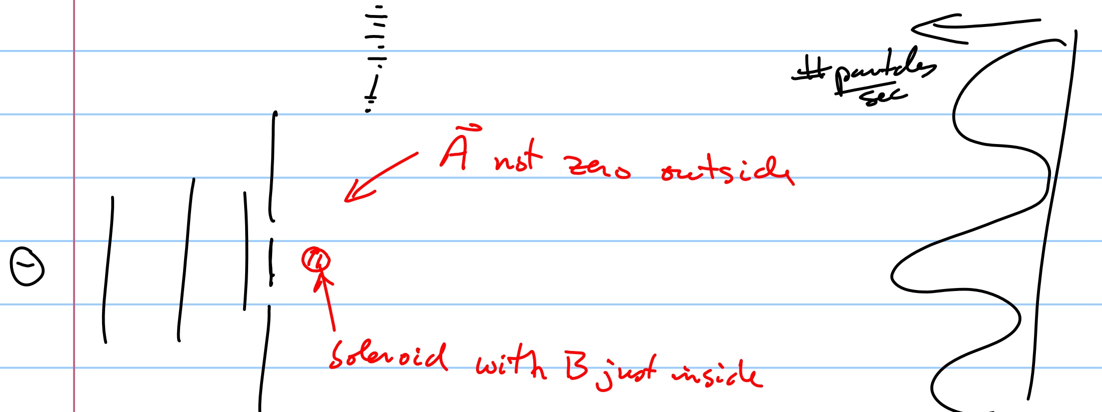
$$\int \underset{B_0}{\vec{B}} \cdot da = \int \nabla \times \vec{A} \cdot da = \oint \vec{A} \cdot d\vec{\ell} = A\, 2\pi r$$

$$B_0 \cancel{\pi} r^2 = |\vec{A}|\, 2\cancel{\pi} r \qquad |\vec{A}| = \frac{B_0 r}{2} \qquad r < R$$

$$B_0 \pi R^2 = |\vec{A}|\, 2\pi r \qquad |\vec{A}| = \frac{B_0 R^2}{2r} \qquad r > R$$







Given $\vec{J}$ find $\vec{A}$

$$\boxed{\nabla^2 V = -\rho/\epsilon_0}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

or $\vec{\nabla}\times\vec{\nabla}\times\vec{A} = \mu_0\vec{J}$

$$\underbrace{\vec{\nabla}(\vec{\nabla}\cdot\vec{A})}_{\substack{= \\ 0}} - \nabla^2\vec{A} = \mu_0\vec{J}$$

Coulomb gauge

vector Laplacian

3 scalar laplace eqn's

$$\boxed{\nabla^2 A_x = -\mu_0 J_x} \qquad \nabla^2 A_y = -\mu_0 J_y \qquad \nabla^2 A_z = -\mu_0 J_z$$

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|}$$

no charge at infinity

$V = \frac{kq}{r}$ $\quad V \to 0$ as $r\to\infty$

$$A_x(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{J_x(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|}$$

$J \to 0$ at ∞