Phys 361 Homework 4

1) Suppose we have a solid, grounded spherical conductor of radius R with no net charge on it, centered at the origin. And suppose we place a positive point charge q on the x axis some distance a from the center of that sphere, with $a > R$. The point charge is going to rearrange the charges in the conductor in some complicated way that has pretty crappy symmetry, making for a situation that's difficult to deal with via standard methods. But the situation *is* vulnerable to method of images.

We're going to find the potential everywhere outside the sphere using images. We can't change the charge in the region where we're finding V, which is all the territory where $r > R$. But we can fiddle with the other territory: the region inside the sphere. Consider an alternate setup where we have the original point charge q at the original location, and also an additional point charge q' at some distance b from the origin, with $b < R$. The charges q, q', and the origin should lie along a straight line, to respect the rotational symmetry of the problem.

a) Write the potential everywhere made by q and q'. It should be the sum of two point charge terms; don't make it to hard.

b) Figure out what b and q' need to be (in terms of other given variables) in order to make it so that the potential is zero for all $r = R$ (where the surface of the grounded sphere would be). A little hint: Let q' and q be on the same side of the origin. Another hint: As far as I can tell, you have to do the algebra in a very particular order to not end up with a trivial solution. Start by factoring an R out of one of the denominators in your voltage expression, and a b out of the other one. That should get you off on a good track.

c) Explain how we can be sure that the potential you wrote in (b) is the correct potential for the original system involving the grounded sphere, at least for $r > R$.

2) Let's get some practice solving Poisson's differential equation without any gimmicks. Find the potential made by a point charge at the origin in empty space. No Gauss's law or Coulomb's law. Do it by writing down Poisson's equation with an appropriate ρ for this system. Then write the boundary conditions for the system. Solve your differential equation, and use the information you have to fix the arbitrary constants and get your solution. Double check that the solution solves Poisson's equation and satisfies your BCs. The final equation for the voltage by itself is, obviously, worth no points.

This one, being a ground-up implementation of one of our most fundamental equations, is probably worth understanding particularly well.

3) (Based on problem 4.20 in Pollack & Stump)

Start with an isolated, solid, spherical conductor of radius *a* and net charge Q_0. Then place it in a uniform electric field of magnitude E_0.

a) Find the complete voltage and field for this situation. It shouldn't be too onerous if you use the example from class plus superposition. Verify explicitly that the E-field satisfies both of the generic E-field boundary conditions from section 3.3.2 (the ones regarding the parallel and perpendicular components of the E-field) at the surface of the conducting sphere.

b) How much additional charge would we need to add to the sphere so that the surface charge density everywhere on it is greater than or equal to zero?

4) Orthogonal functions are a pretty big deal when it comes time to solve PDEs. Some of our best series solution techniques absolutely require orthogonal sets, so let's work on getting some intuitive feel for how orthogonality can work with *functions* (you should already have an intuitive understanding of orthogonal *vectors*).

a) We already know that $\sin\frac{j\pi y}{L}$ and $\sin\frac{n\pi y}{L}$ are orthogonal* over the interval 0 to L if $j \neq n$ and not orthogonal if $j = n$. Explain in words and sketches why this should be so.

b) Now let's generalize things a bit. Consider the interval –L to L for some arbitrary L. Use the definition of orthogonality and direct integration to find all L for which $\sin x$ and $\cos x$ would be orthogonal on that interval. Then explain in hindsight why you might have been able to see the result coming.

c) What about $\sinh x$and $\cosh x$? Are they orthogonal on the interval 0 to L? How about from –L to L? You may either use sketches or integration. Don't sweat it too hard; just be decently convincing.

d) Finally, how about x^3 and $\cosh x$ from –L and L?

e) Can you make any general statements about the orthogonality of functions based on the previous? There's plenty you could say here, so in addition to whatever else you might say, try to work in a statement involving parity (even/oddness).

*Just to make sure we're on the same page, two distinct, real-valued functions *f(x)* and *g(x)* are orthogonal on the interval from *a* to *b* if

$$\int_a^b f(x)g(x)dx = 0$$