Problem 1:

Recall that

$$\text{Conservation of Stuff} \Rightarrow \frac{\partial u}{\partial t} - f + \operatorname{div}(\vec{\Phi}) = 0$$

Where

- $[f] \equiv \dfrac{\text{Stuff}}{\text{Volume}\cdot\text{time}}$, is a $\dfrac{\text{stuff}}{\text{Volume}}$ generation term
- $[u] \equiv \dfrac{\text{Stuff}}{\text{Volume}}$, is a stuff density
- $[\vec{\Phi}] \equiv \dfrac{\text{Stuff}}{\text{Area}\cdot\text{time}}$, is a stuff flux

In 1D with no source we have

$$(1)\quad \frac{\partial u}{\partial t} + \frac{\partial \Phi}{\partial x} = 0 \Rightarrow \frac{\partial u}{\partial t} = -\frac{\partial \Phi}{\partial x}$$

Note: The homework has a constant but there are many to be had. It won't matter

1.1: Recall for when stuff is heat Energy then Fourier says

$$\vec{\Phi} = -D\vec{\nabla}u = -D\frac{\partial u}{\partial x}, \text{ in 1D,}$$

Where D is a diffusivity.

We now assume, instead of Fourier's law, that flux is Related to density
$\underbrace{}_{\text{Proportional}}$

$$\Rightarrow \quad \phi \propto u \Rightarrow \phi = cu, \text{ for some constant } c \in \mathbb{R}.$$

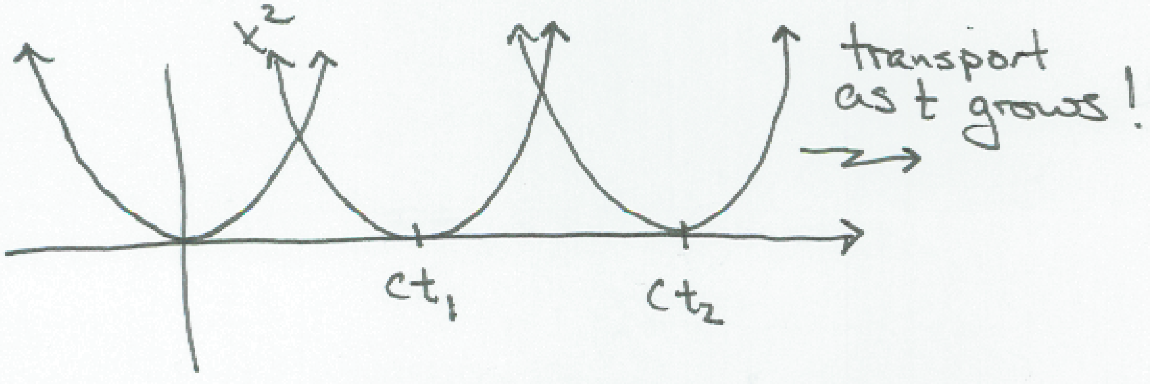
Thus (1) becomes

$$(*) \quad \frac{\partial u}{\partial t} + \frac{\partial \phi}{\partial x} = \frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left[cu\right] = \frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0,$$

Which is not a heat Eqn. In fact one can show that

$$u(x,t) = f(x-ct)$$

is a general Soln to (*). What does this soln mean? We if $f(x) = x^2$ then below we have $f(x-ct)$



The important point is that the datum is transported without deformation! Thus, by making a different assumption on ϕ we get a new PDE whose soln has different character than soln to heat Eqn, which spread and Equalize over time.

This may be thought of as waves coming up from the hot coffee cup as opposed to the diffusive spread of the coffee's heat blob.

1.3: $\phi = \alpha U_x + \beta U$ assumes both convection + diffusion.

1.4: Assumes

$$U_t = DU_{xx} + CU_x + U$$

Where

$$U_t = U$$

Represents growth or decay of the stuff.

1.5: Notes:

- watch out for the footnote, it is $\beta = \lambda + c^2/(4D)$ **not** $\beta = \lambda + c^2/(4D)^4$

- watch for product and chain Rules!

- $U_t = W_t e^{\alpha x - \beta t} - w\beta e^{\alpha x - \beta t}$

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The key point here is that if you have the other terms present for convection + decay then the mathematical problem is again the heat Eq.