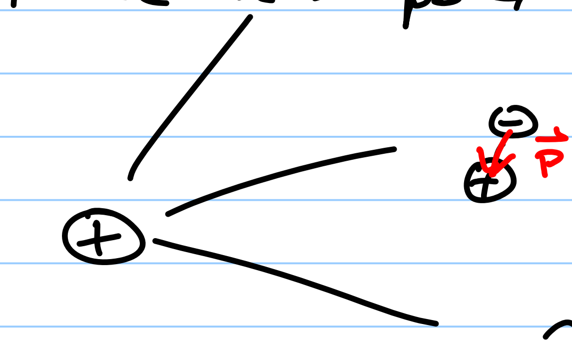
Force on dipole



$$\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$$

$$E_x(x,y,z) \qquad E_y(x,y,z)$$

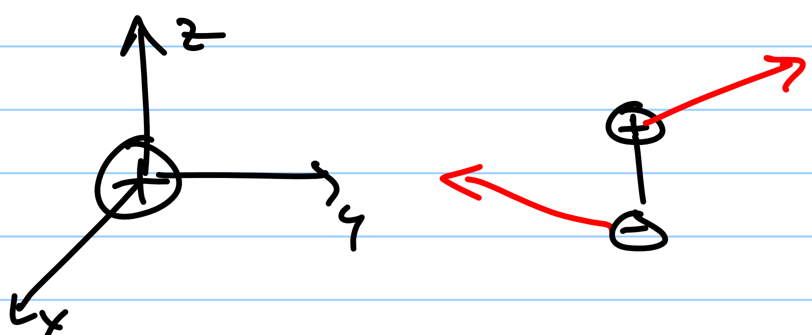
$$\vec{F} = \vec{F}_+ + \vec{F}_- = q\big(\overbrace{\vec{E}_+ - \vec{E}_-}^{d\vec{E}}\big) = q\big(\underbrace{dE_x}_{\frac{\partial E_x}{\partial x}dx + \frac{\partial E_x}{\partial y}dy + \frac{\partial E_x}{\partial z}dz}\hat{x} + dE_y\hat{y} + dE_z\hat{z}\big)$$

$$dE_x = \underbrace{\left(\frac{\partial E_x}{\partial x}\hat{x} + \frac{\partial E_x}{\partial y}\hat{y} + \frac{\partial E_x}{\partial z}\hat{z}\right)}_{\vec{\nabla}E_x} \cdot \underbrace{\left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right)}_{d\vec{r} \Leftarrow \text{charge sep.} = \vec{s}}$$

$$\vec{\nabla}E_x \cdot \vec{s}$$

$$d\vec{E} = (\vec{s}\cdot\vec{\nabla})\vec{E}$$

$$\vec{F} = q\,d\vec{E} = (\vec{p}\cdot\vec{\nabla})\vec{E}$$

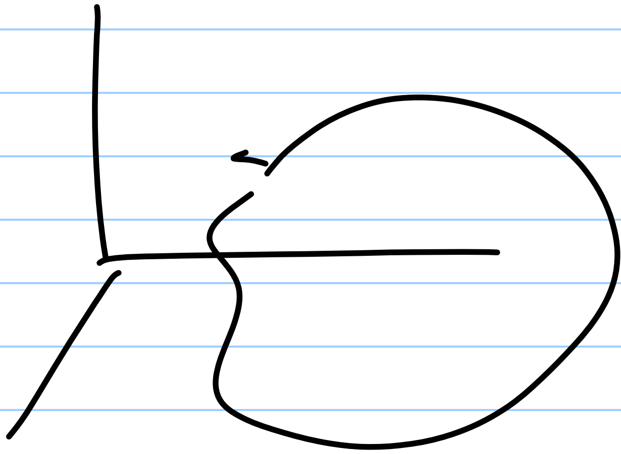


$$\vec{p} = p_0\hat{z}$$

$$\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \qquad \vec{\mathscr{r}} = \underbrace{\vec{r}}_{x\hat{x}+y\hat{y}+z\hat{z}} - \underbrace{\vec{r}\,'}_{0}$$

$$p_0\hat{z}\cdot\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\frac{1}{4\pi\epsilon_0}\frac{q\,(x\hat{x}+y\hat{y}+z\hat{z})}{(x^2+y^2+z^2)^{3/2}}$$

collection of dipoles: $\vec{P}$ $\frac{\text{dipole mom}}{\text{vol}}$



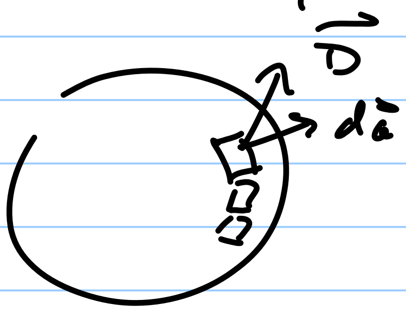
$$\sigma_b = \vec{P}\cdot\hat{n}$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

Gauss's Law $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_b + \rho_f}{\epsilon_0}$

$$\vec{\nabla}\cdot(\underbrace{\epsilon_0\vec{E} + \vec{P}}_{\vec{D}}) = \rho_f$$

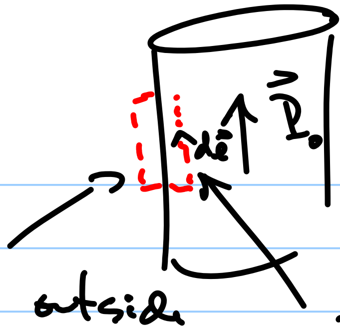
$$\vec{\nabla}\cdot\vec{D} = \rho_f \qquad \int \vec{\nabla}\cdot\vec{D}\,d\tau = \int \rho_f\,d\tau$$



$$\oint \vec{D}\cdot d\vec{a} = Q_{free}$$

To define a vector field uniquely you need $\vec{\nabla}\cdot\vec{D}$ & $\vec{\nabla}\times\vec{D}$

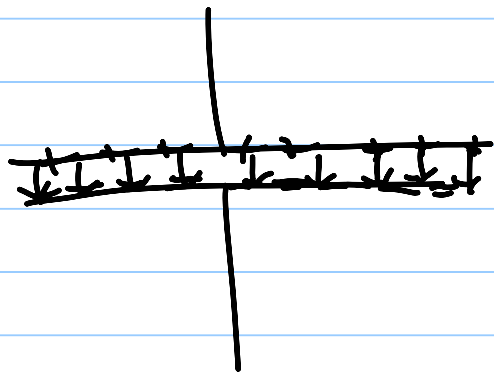
$$\vec{\nabla}\times\vec{D} = \vec{\nabla}\times(\epsilon_0\vec{E} + \vec{P}) = \epsilon_0\underbrace{\vec{\nabla}\times\vec{E}}_{0} + \underbrace{\vec{\nabla}\times\vec{P}}_{?}$$

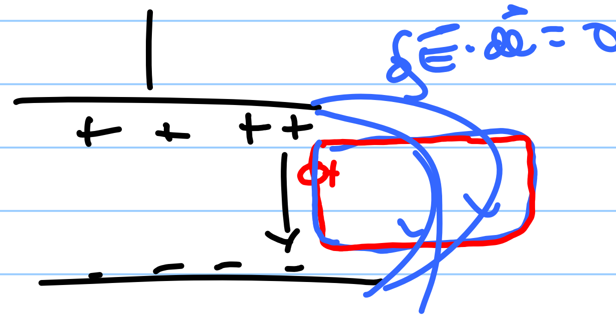


$$\int \vec\nabla\times\vec P\cdot d\vec a = \oint \vec P\cdot d\vec l$$

$\vec P\cdot d\vec l = 0$ outside

$P\cdot dl \neq 0$ inside





$$\vec\nabla\times\vec E = 0$$

$$\int \underbrace{\vec\nabla\times\vec E}_{0}\cdot d\vec a = \oint \vec E\cdot d\vec l = 0$$