velocity equal to that slope (equation 67.4). The wave velocity can thus be positive or negative! In the next section we will attempt to describe what a density wave is, in particular, what it means for the velocity of a density wave to be negative!

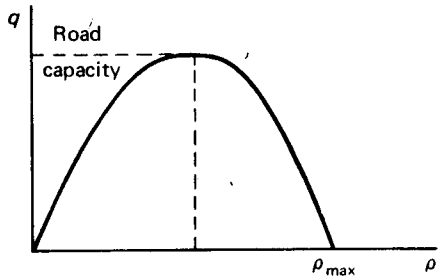

Figure 67-5 Flow-density curve: road capacity.

EXERCISES

67.1. Suppose that

$$u = u_{max}\left(1 - \frac{\rho}{\rho_{max}}\right).$$

(a) What is the density wave velocity if the traffic density is nearly ρ_0 everywhere?
(b) Show that this density wave velocity is less than the car's velocity.

67.2. Suppose initially ($t = 0$) that the traffic density is

$$\rho = \begin{cases} \rho_0 & \text{if } |x| > a \\ \rho_0 + \epsilon & \text{if } |x| < a, \quad \text{where } |\epsilon| \ll \rho_0. \end{cases}$$

(a) Sketch $\rho(x)$ at $t = 0$.
(b) Sketch $\rho(x)$ at $t = 2$.

67.3. Suppose initially ($t = 0$) that the traffic density is $\rho = \rho_0 + \epsilon \sin x$, where $|\epsilon| \ll \rho_0$. Determine $\rho(x, t)$.

67.4. Consider $\partial\rho_1/\partial t + c(\partial\rho_1/\partial x) = 0$. Suppose we observe ρ_1 in a coordinate system moving at velocity v. Show that

$$\frac{\partial\rho_1}{\partial t'} + (c - v)\frac{\partial\rho_1}{\partial x'} = 0.$$

Does ρ stay constant moving at the car velocity?

67.5. Based on a linear analysis, would you say $\rho = \rho_0$, a constant, is a stable or unstable equilibrium solution of equation 66.1?

67.6. Show that $c = dq/d\rho\ (\rho_0)$ has the dimensions of a velocity.

67.7. What is the *slope* of the straight line characteristics sketched in Fig. 67-3.

68. Traffic Density Waves

For convenience **heavy traffic** is defined as traffic such that the density is greater than the optimal density (corresponding to the road's capacity) and **light traffic** such that the density is less than the optimal density; see Fig. 68-1. Using these definitions and the Fundamental Diagram of Road Traffic, we conclude that in heavy traffic the perturbed densities move with a negative velocity, while in light traffic the opposite is true.

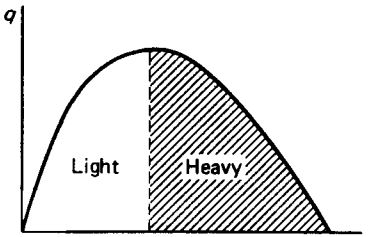

Figure 68-1 Light and heavy traffic.

Let us imagine a situation involving *heavy traffic of a nearly uniform density*. Suppose that the initial density is sketched in Fig. 68-2, where the dotted line is the approximate constant initial density, and • indicates a relative minimum or relative maximum of the density. The previous analysis has shown that the density remains constant if an observer moves with velocity c which is negative. Thus density is constant along characteristics, sketched in a space-time diagram, Fig. 68-3. Positions of the relative maxima

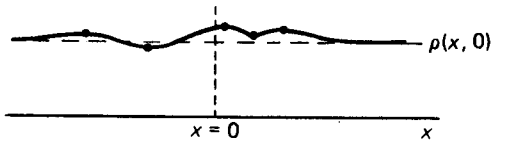

Figure 68-2 Nearly uniform heavy traffic.

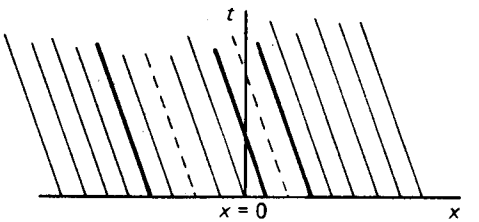

Figure 68-3 Characteristics of $\partial\rho_1/\partial t + c\,\partial\rho_1/\partial x = 0$.