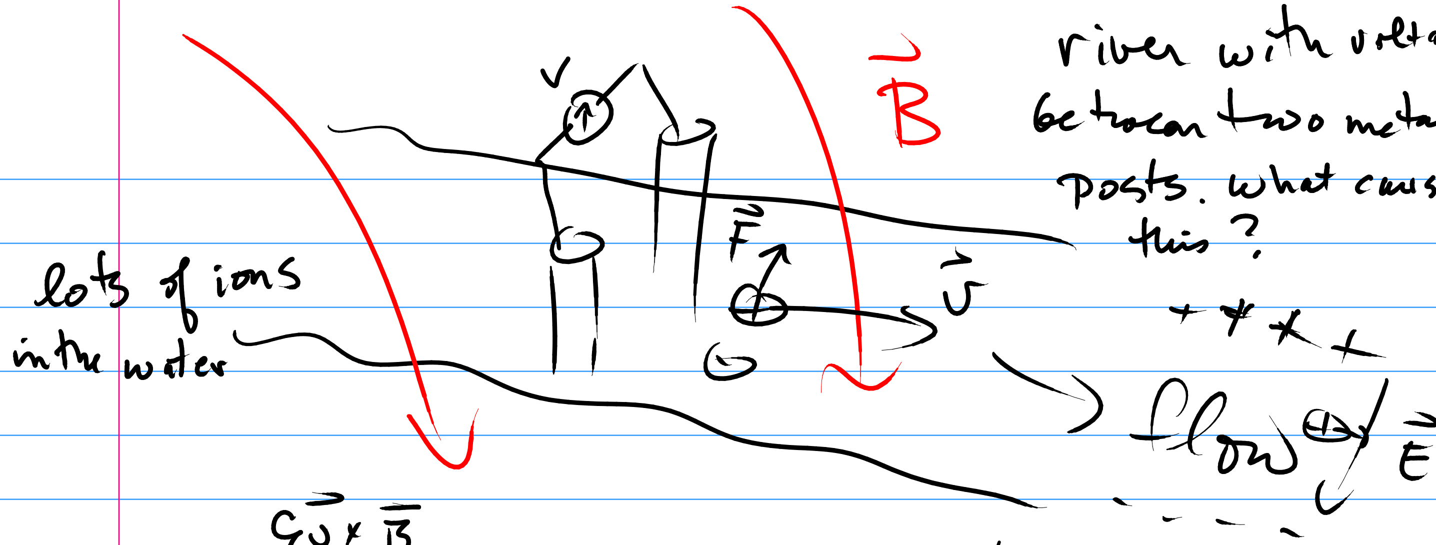


river with voltage between two metal posts. what causes this?

$q\vec{v}\times\vec{B}$

$qE = qvB$

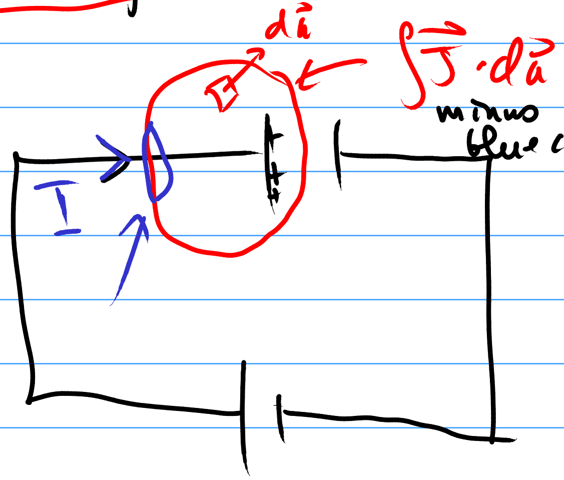
Maxwell's Equations

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

for magnetostatics

$$\vec{\nabla}\cdot\vec{J} = 0 \quad \left(-\frac{\partial\rho}{\partial t}\right)$$



$\oint\vec{J}\cdot d\vec{a} = 0$

minus blue cap

$\int\vec{J}\cdot d\vec{a} = \mu_0 I$

$$\int\vec{\nabla}\cdot\vec{J}\,d\tau = \oint\vec{J}\cdot d\vec{a}$$

$$\int\vec{J}\cdot d\vec{a} + \int\vec{J}\cdot d\vec{a} = 0$$

blue cap $= \mu_0 I$ + rest $= 0$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} \qquad \underbrace{\int \vec{\nabla}\cdot\vec{J}\, d\tau}_{\oint \vec{J}\cdot d\vec{a}} = -\frac{\partial}{\partial t}\underbrace{\int \rho\, d\tau}_{Q_{enc}}$$

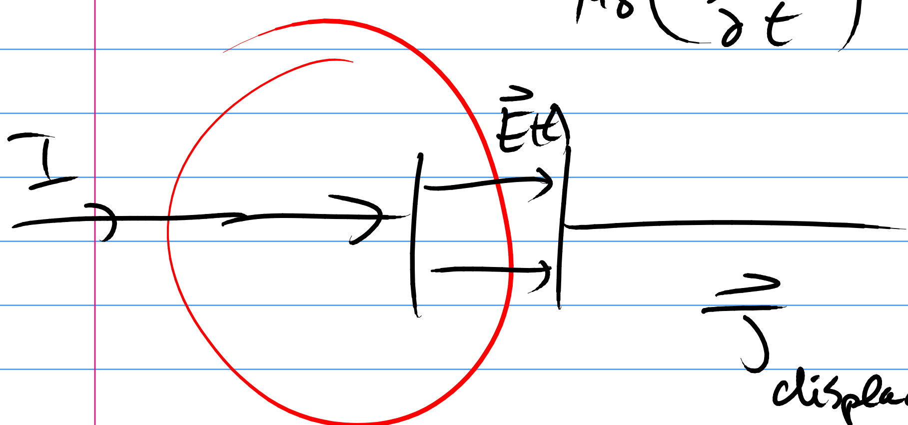
$$\oint \vec{J}\cdot d\vec{a} = -\frac{d}{dt} Q_{enc}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right) = 0$$

$$\vec{\nabla}\cdot\mu_0\vec{J} = \mu_0\,\vec{\nabla}\cdot\vec{J} \neq 0$$

$$\mu_0\left(-\frac{\partial\rho}{\partial t}\right)$$



$$\oint \vec{J}\cdot d\vec{a} = -\frac{dQ_{enc}}{dt}$$

$$\vec{J}_{displacement} = \epsilon_0 \frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \left(\mu_0\vec{J} + \mu_0\underbrace{\epsilon_0\frac{\partial\vec{E}}{\partial t}}_{J_D}\right) \qquad \vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

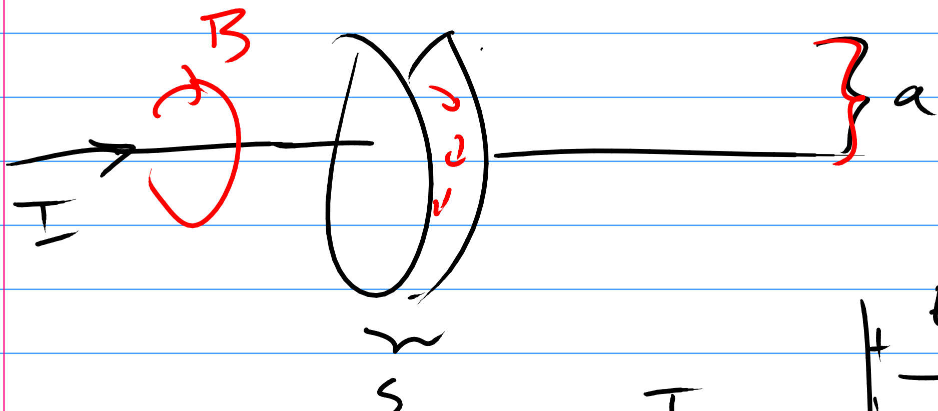
$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right) = 0$ always

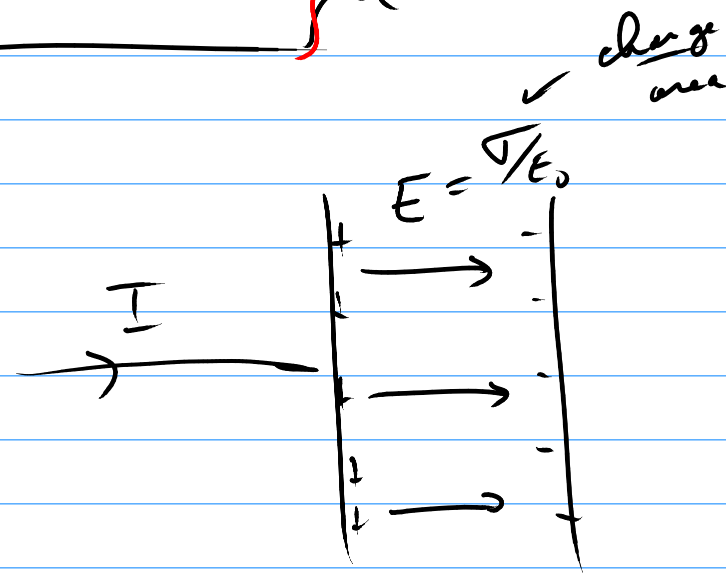
$$\vec{\nabla}\cdot\left(\mu_0\vec{J}+\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)=\mu_0\underbrace{\vec{\nabla}\cdot\vec{J}}_{=}+\mu_0\epsilon_0\frac{\partial}{\partial t}\underbrace{\vec{\nabla}\cdot\vec{E}}_{=\rho/\epsilon_0}$$

$$\mu_0\left(-\frac{\partial\rho}{\partial t}\right)+\mu_0\frac{\partial\rho}{\partial t}=0$$

General form for ME.

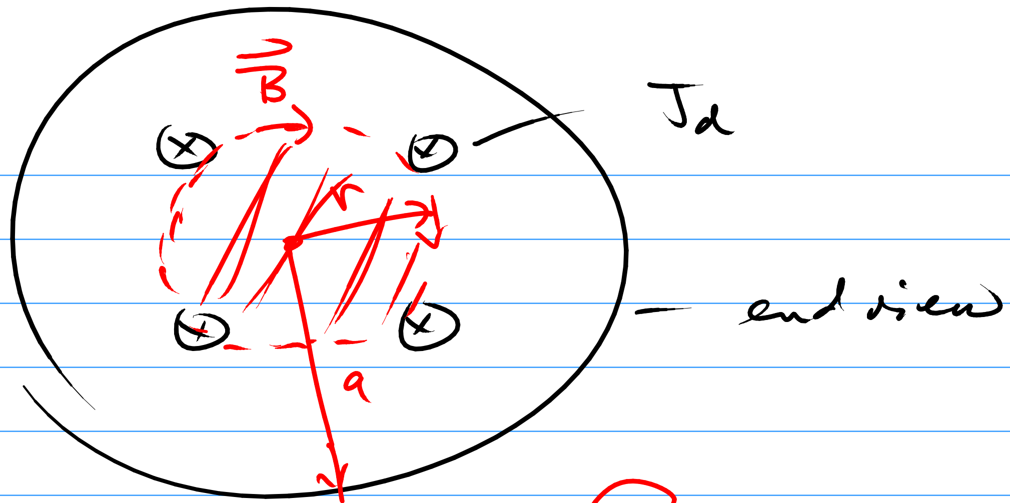
$$\vec{\nabla}\times\vec{B}=\left(\mu_0\vec{J}+\mu_0\underbrace{\epsilon_0\frac{\partial\vec{E}}{\partial t}}_{J_D}\right)$$





$$E=\frac{\sigma}{\epsilon_0}=\frac{Q(t)}{\epsilon_0\pi a^2}\qquad\frac{\partial E}{\partial t}=\frac{1}{\epsilon_0\pi a^2}\underbrace{\frac{dQ(t)}{dt}}_{I}$$

$$J_{\text{displacement}}=\epsilon_0\frac{\partial E}{\partial t}=\frac{1}{\pi a^2}I(t)$$



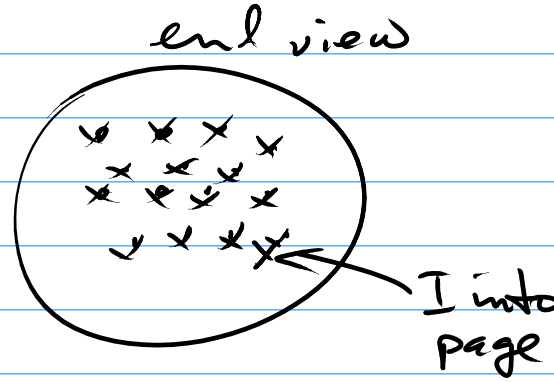
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} = \mu_0 \int \vec{J}_d \cdot d\vec{a}$$

$$\oint |\vec{B}||d\vec{\ell}| \underbrace{\cos 0}_{1} = B\, 2\pi r = \mu_0 \frac{I(t)}{\cancel{\pi} a^2} \cancel{\pi} r^2$$

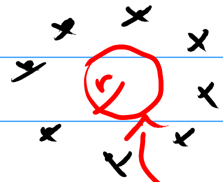
$$B \cancel{2\pi r} = \mu_0 \frac{I(t)\, r^{\cancel{2}}}{2\pi a^2}$$

Class question. Consider a wire composed of smaller wires.

Doesn't B cancel inside due to the other wires?



Cons: consider a ring of such wires



B inside the ring is 0 by ampere's law. Choose amperian path. B is same along path due to symmetry so

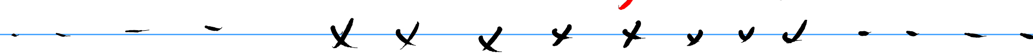
$$\oint \vec{B}\cdot d\vec{\ell} = 2\pi r B = \mu_0 \underset{\overset{\shortparallel}{0}}{I_{enclosed}}$$

So $B = 0$

However for this path outside the ring $B \neq 0$.

why? Consider an analogy with sheet of current

$\longrightarrow B_0$ } Here B adds
$\longrightarrow$

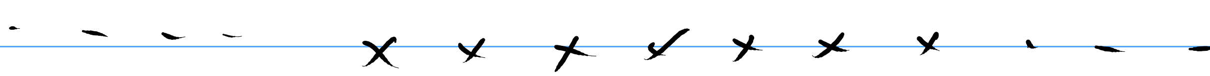


$\longleftarrow B_0$ } Here B cancels
$\longrightarrow B_0$

$\longleftarrow B_0$ } Here B adds
$\longleftarrow B_0$

So the net B is

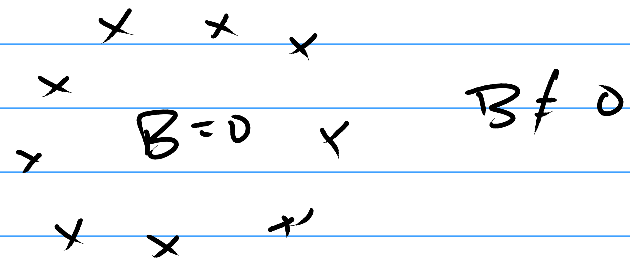
$\longrightarrow$ $B = 2B_0$



$B = 0$

$\longleftarrow$ $B = 2B_0$

Now fold these two sheets together to form a circle



In a cylinder filled with wires

we can order them into circles of wires each of which has $B = 0$ inside. Note that B will only be zero on the symmetry axis.