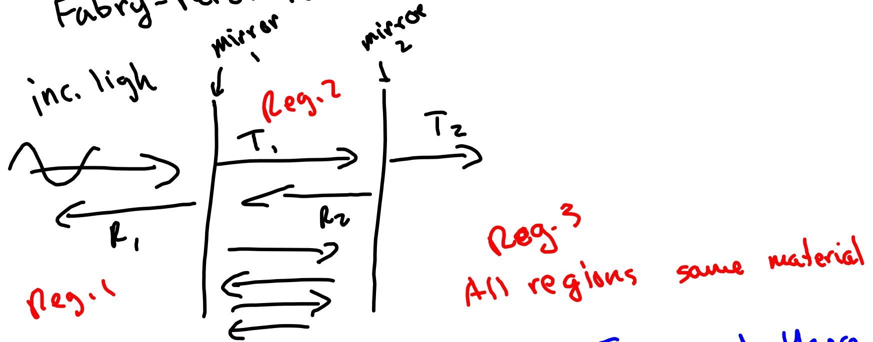
Resonators

Resonances: A resonance is when you have a wave that bounces back & forth in some way where subsequent reflections constructively interfere.

Let's look at the simplest optical resonator Fabry-Perot resonator (interferometer).



Say inc. electric field is E_0 and there are no losses in mirrors. Each mirror has a reflection coefficient of R...

so $R + T = 1$
transmission

$$R = \frac{I_R}{I_{inc}} = \frac{E_R^2}{E_0^2}$$

$$T = \frac{I_T}{I_{inc}} = \frac{E_T^2}{E_0^2}$$ only because materials are same.

I'd like to do stuff in terms of E so let's define $r = \sqrt{R} = \frac{E_R}{E_0}$; $t = \sqrt{T} = \frac{E_T}{E_0}$

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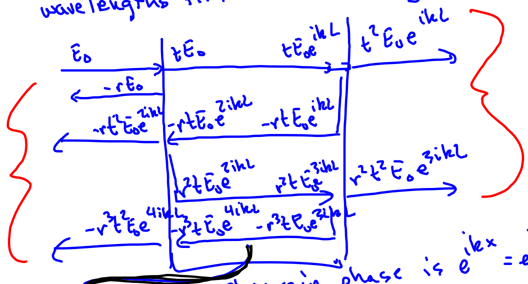
If something is on resonance then in this type of cavity an $\frac{1}{2}$integer # of wavelengths fits in the cavity.



For the inside, use the center.

Change in phase is $e^{ikx} = e^{ik\Delta x}$

For arbitrary lengths, rather than $(-1)^n$, you would do e^{ikmL}

# of reflection

Write out the infinite series for the reflection, transmission, and E_{inside}

Reflection:

$$-rE_0 - rt^2E_0e^{2ikL} - r^3t^2E_0e^{i4kL}$$

$$-rE_0 - rt^2E_0e^{2ikL}\sum_{n=0}^{\infty}(r^2e^{2ikL})^n$$

$$= -rE_0 - rt^2E_0e^{2ikL}\left(\frac{1}{1-r^2e^{2ikL}}\right)$$

$$= -E_0r\left(1 + \frac{t^2e^{2ikL}}{1-r^2e^{2ikL}}\right)$$

This should be minus but I can't figure out why...

Transmission:

$$t^2E_0e^{ikL} + r^2t^2E_0e^{i3kL} + \ldots$$

$$E_0t^2e^{ikL}\sum_{n=0}^{\infty}(r^2e^{2ikL})^n$$

$$= \frac{E_0t^2e^{ikL}}{1-r^2e^{2ikL}}$$

Inside: $tE_0e^{ikL/2} - rtE_0e^{ik3L/2} + r^2tE_0e^{ik5L/2} + \ldots$

$$E_0te^{ikL/2}\sum_{n=0}^{\infty}(-re^{ikL})^n = \frac{E_0te^{ikL/2}}{1+re^{ikL}}$$

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$$\text{Transmission} = \frac{|E_T|^2}{E_0^2} = \left| \frac{E_0 t^2 e^{ikL}}{1 - r^2 e^{2ikL}} \right|^2 \Big/ E_0^2$$

$$= \left| \frac{T e^{ikL}}{1 - R e^{2ikL}} \right|^2$$

$$= \frac{(1-R)^2}{1 + R^2 + R\left(e^{2ikL} + e^{-2ikL}\right)}$$

$$\boxed{T_{tot} = \frac{(1-R)^2}{1 + R^2 + 2R\cos(2kL)}}$$

Intensity inside:

$$E_{inside} = \frac{E_0 t e^{ikL/2}}{1 + r e^{ikL}} \rightarrow T_{tot} = \frac{|E_i|^2}{E_0^2} = \frac{t^2}{1 + r^2 + r\left(e^{ikL} + e^{-ikL}\right)}$$

$$\rightarrow \boxed{\frac{I_{inside}}{I_{inc}} = \frac{1-R}{1 + R + 2\sqrt{R}\cos(kL)}}$$

Reflection:

$$E_r = -E_0 r \left(1 - \frac{t^2 e^{2ikL}}{1 - r^2 e^{2ikL}} \right)$$

$$= -E_0 \sqrt{R} \left(\frac{1 - R e^{2ikL} - (1-R) e^{2ikL}}{1 - R e^{2ikL}} \right)$$

$$= -E_0 \sqrt{R} \left(\frac{1 - e^{2ikL}}{1 - R e^{2ikL}} \right)$$

$$\rightarrow \boxed{R_{tot} = \frac{|E_r|^2}{E_0^2} = R \left(\frac{2 - 2\cos(2kL)}{1 + R^2 - 2R\cos(2kL)} \right)}$$

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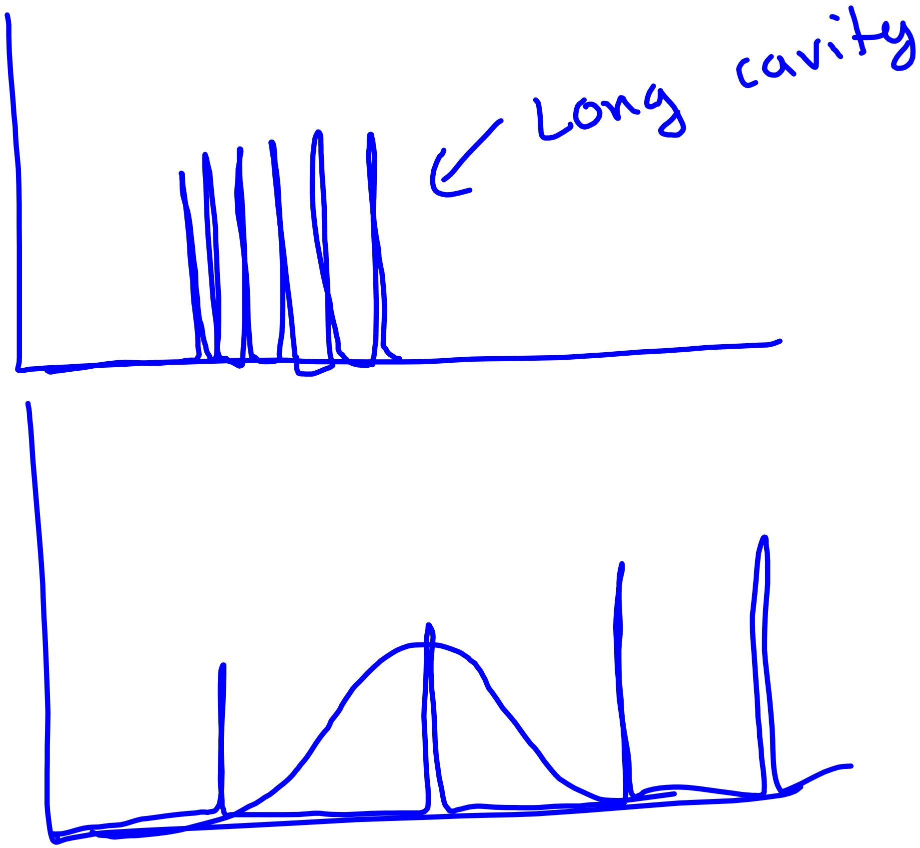
Transmission

Lasers



Changing mirror quality