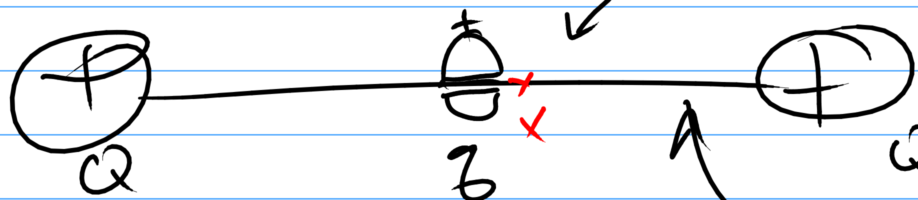
$$PE = \left(\frac{kQ}{r_1^2}\hat{r}_1 + \frac{kQ}{r_2^2}\hat{r}_2\right)q$$

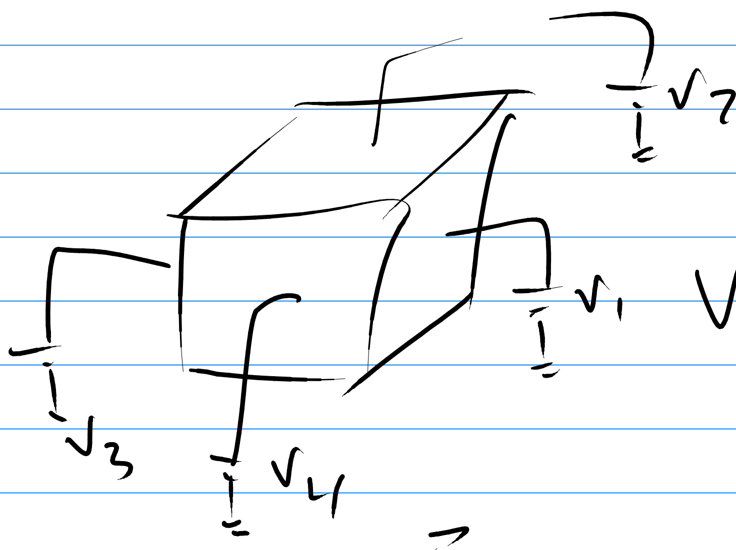
$\nabla^2 V = 0$ 3-D problem



$$V = \sum_i v_i$$

$$V = X(x)\,Y(y)\,Z(z)$$

const



$$V = X(x)\,Y(y)\,Z(z)$$

∞ parallel plates

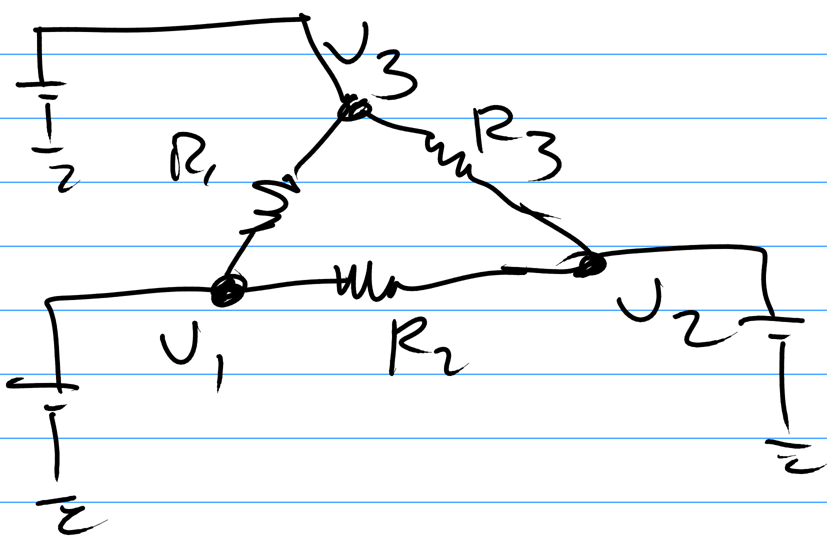
$$\frac{d^2 Y}{dy^2} =$$

$$V = Y(y)$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$$

$$C_1 + C_2 + C_3$$

Superposition Principle



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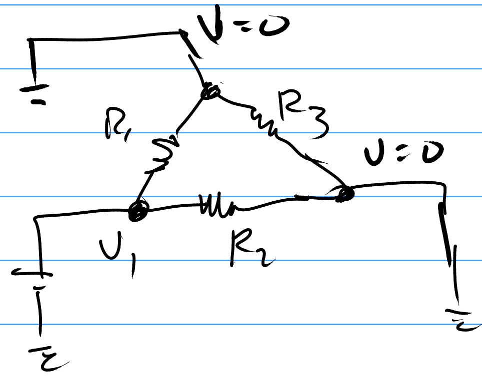

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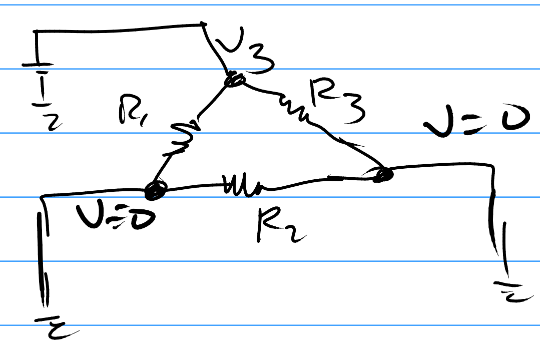

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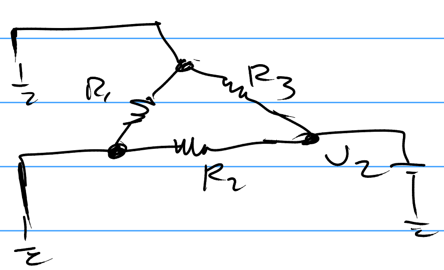


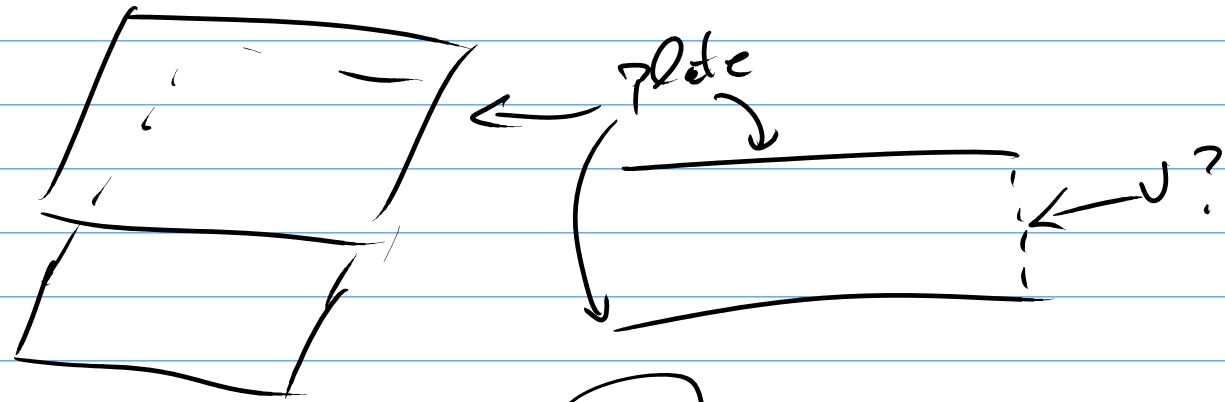
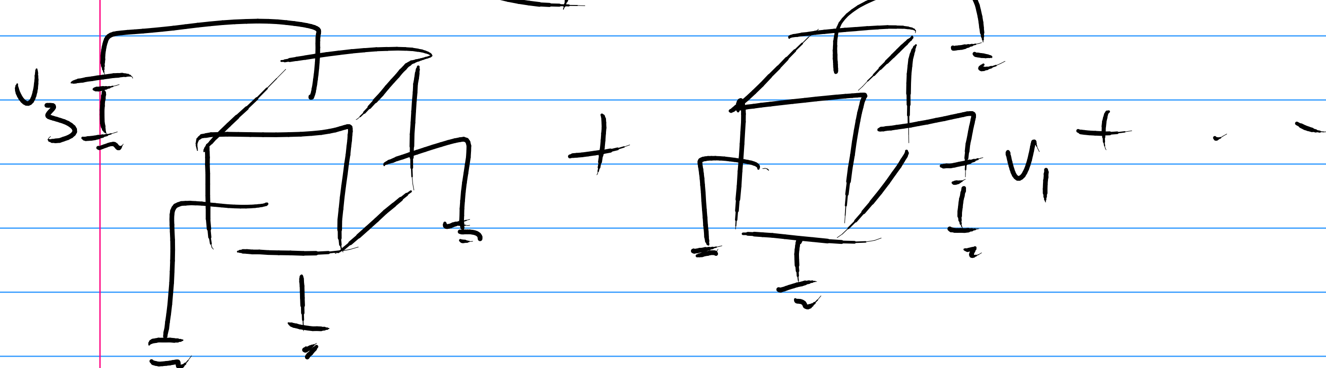
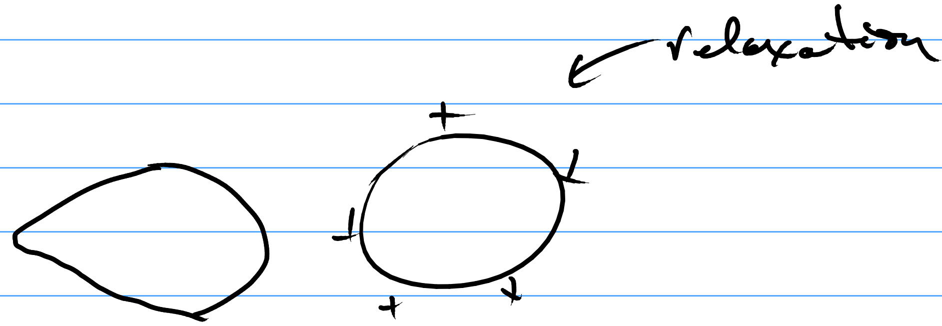
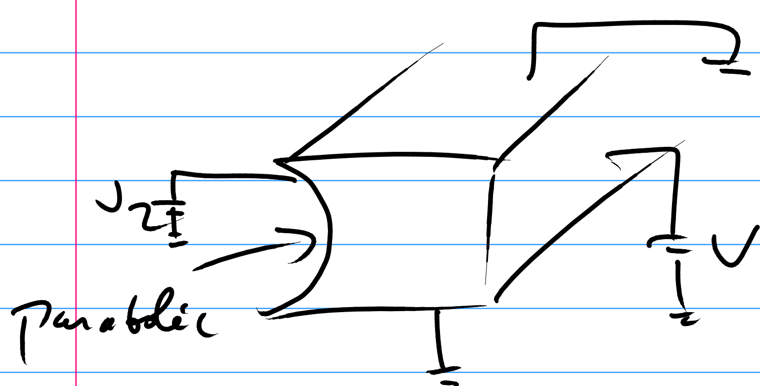
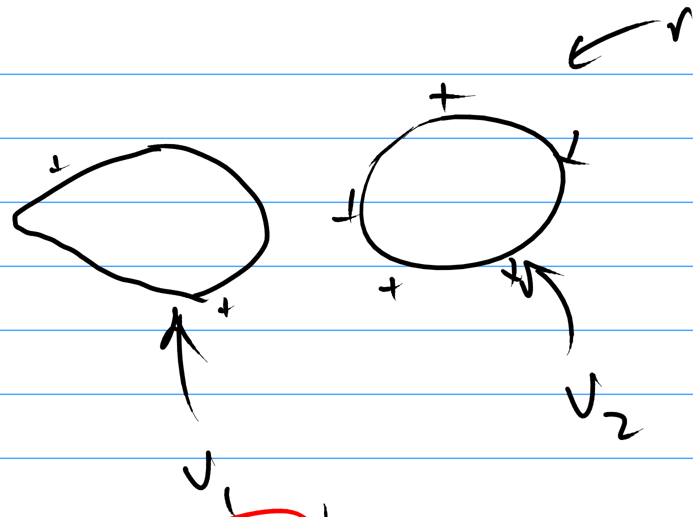


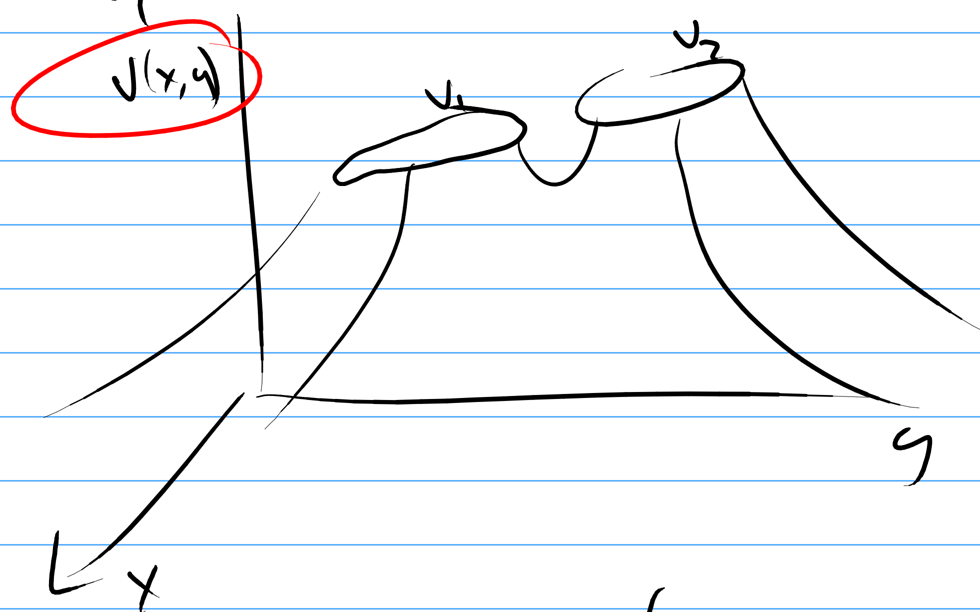
plate
V ?


V_3
+
V_1
+ . . .


relaxation


V_2
V
Parabolic





Energy of this system is given by

$$\vec{E} = -\vec{\nabla} V$$

Energy to assemble charge distribution

$$E_{tot} = \int \frac{1}{2}\epsilon_0 E^2 \, d\tau \qquad \vec{\nabla} V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\vec{E}\cdot\vec{E} = +\vec{\nabla} V \cdot \vec{\nabla} V = \left(\frac{\partial V}{\partial x}\right)\left(\frac{\partial V}{\partial x}\right) + \cdots$$

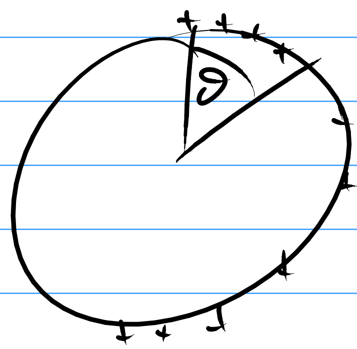
Find E_{tot} for $V_1 = 10$ $V_2 = 12$

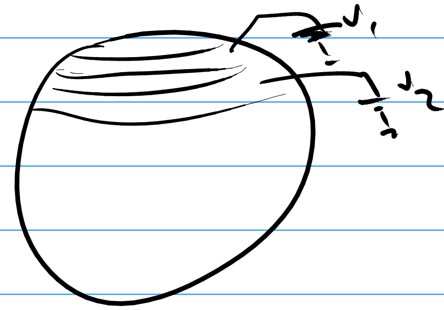
" " " $V_1 = 11$ $V_2 = 12$

$\nabla^2 V = 0$ in spherical coords

$V(r,\theta,\varphi) = R(r)\,\Theta(\theta)\,\Phi(\varphi)$

NO φ dependence $V(\theta)$



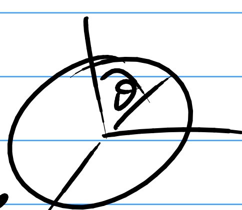


$$V = \sum_{l=0}^{\infty} \left(A_l r^l + B_l r^{-(l+1)}\right) P_l(\cos\theta)$$

$$\int_{-1}^{1} P_l(x)\,P_m(x)\,dx = \int_0^{\pi} P_l(\cos\theta)\,P_m(\cos\theta)\,\underbrace{\sin\theta\,d\theta}_{dx}$$

$V(r<R) = \sum_l A_l P_l$

$V(r>R) = \sum_l B_l P_l$



$V(\theta)$ at $r = R$

$$= \begin{cases} 0 & \text{if } l \neq m \\ \frac{2}{2m+1} & \text{if } l = m \end{cases}$$