Physical and Fourier Optics (PHGN570) posted 12 January 2011
Homework 1 due in class 19 January 2011

1. Consider a pulse of the form
$f(t) = Ae^{-|t|/b}e^{-i\omega_0 t}$
where b is a real, positive constant.
   a. Calculate the Fourier transform $F(\omega) = \Im\{f(t)\}$ by direct integration, manually.
   b. Do this transform using the FourierTransform[ ] function in Mathematica. Our convention for the transforms requires you use the options FourierParameters $\rightarrow \{1,1\}$.
   c. Let $l(t) = e^{-t}$ for $t > 0$, and $= 0$ for $t < 0$. Calculate the Fourier transform $L(\omega)$ of the decaying exponential function, l. Express $f(t)$ in terms of $l(t)$, and use Fourier identities to calculate $F(\omega)$ (no additional integration needed).
   d. Show that for this pulse $\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$ by calculating both sides of the equation, confirming Parseval's theorem. You may use Mathematica for this.

2. Given that the Fourier transform of *f(t)* is *F(ω)*, find general expressions for the transforms of $g(t) = \int_0^t f(t')dt'$ and $h(t) = df/dt$.

3. Consider a laser pulse that has a Gaussian temporal shape. The pulse has a center wavelength of 800nm. For several transform-limited pulse durations, calculate the width of the spectrum in angular frequency ($\Delta\omega$) and in wavelength ($\Delta\lambda$). In experimental work, we specify all the widths in terms of the full-width at half maximum (FWHM) of the intensity profile. Do the calculation of $\Delta\omega$ and $\Delta\lambda$ for τ_{fwhm} = 3fs, 20fs, 100fs and 1ps:
   - you will be able to calculate $\Delta\omega$ analytically from the transform
   - for $\Delta\lambda$ you will need to either solve for the half-power points after converting the profile to wavelength, or the get the widths from a plot
   - To illustrate the distortion in λ space, plot the spectra for the 3fs pulse in both ω and λ space. Comment on the difference in shape.
   - A common way to convert spectral widths is to use the relation $\Delta\omega/\omega_0 = \Delta\lambda/\lambda_0$. This is a very good rule of thumb only if $\Delta\omega/\omega_0 \ll 1$. Calculate $\Delta\lambda$ from $\Delta\omega$ and compare to your more exact results above.
4. Calculate the Fourier transform of a triangle pulse: $f(t) = 1 - |t| \quad (-1 < t < 1)$.
You can either do the integral directly or relate the triangle pulse to the autoconvolution of a square pulse.