PHGN 462 Homework 5

1) (From Pollack and Stump 14.1)

For the TE(n) mode of propagation in the space between parallel conducting planes, show that $\vec{S}_{avg} = u_{avg} v_{gr} \hat{z}$, where u is the field energy density. Here the subscript avg implies an average over both t and y. Explain why this result means that the group velocity is the velocity of energy transport.

2) (From Pollack and Stump 14.6) This is supposed to be short and plug 'n chug. Don't make it harder than it is.

Consider a car entering a tunnel of dimensions 15 m wide and 4 m high. Assuming the walls are good conductors, can AM radio waves propagate in the tunnel?

3) We're spending a lot of time talking about how E&M waves propagate in various waveguides, and not a lot of time thinking about why we bother to propagate them. Root around in books or on the interwebs and learn about the actual applications of microwave waveguides. Write up a short description of two different applications. It would be nice if at least one of them came into common usage after 1960 (a lot of microwave tech is pretty old).

And don't take advantage of this problem by copying and pasting stuff from, say, the first and second websites that come up when you search for "microwave waveguides." I know which sites those are. I also know what the third and fourth hits are, before you get any clever ideas.

4) One point I try to make in junior-level classes is that special functions aren't all that special. Cosines, sines, Legendre polynomials, Bessel functions; it's all the same stuff in the end. Orthogonal functions that solve a certain differential equation and that have a bunch of identities associated with them that you can find in tables. This problem uses Bessel functions. Keep calm.

Anyway, let's think about a hollow conducting cylindrical waveguide of radius a (this is not the same as the coaxial pair of cylindrical shells that we did/will do in class… just one cylinder). Let the axis of the cylinder be in the $\hat{k}$ direction.

a) There are a lot of different ways to construct solutions to a new geometry. But easiest is to take a look at the general problem that isn't specific to any geometry. Section 14.3 in Pollack and Stump does this, so read that (or the equivalent) up until eqn 14.74: $\nabla^2\psi = -\gamma^2\psi$. That equation should look pretty familiar, because we've done this before: We solved the general problem and then made it a specific problem by applying rectangular boundary conditions. But this time we're going to apply cylindrical boundary conditions, so solve 14.74 using separation of variables in cylindrical coordinates (technically polar since we're in 2D).

b) If you start going after the above with separation of variables, you should end up with a fairly trivial equation for φ and Bessel's equation for γr, where $\gamma^2 = \left(\frac{\omega}{c}\right)^2 - k^2$. Don't panic! Just read about Bessel's equation and the solutions to it. We're not really going to have to do all that much with them. Put everything together and write down the solution for ψ and then for the magnetic field. The solution should be indexed by some integer; let's call it m. Use a complex exponential for the φ equation instead of sines and cosines.

c) Demonstrate how to find the parameter γ corresponding to a particular mode m. With that, you can complete the dispersion relation. Unless you're very clever, you'll probably at some point have to say "And here's where we can't proceed analytically anymore, so numbers." And that's fine. Sometimes that really is the answer.

5) (From Pollack and Stump 14.8)

a) Consider a wave guide with a square cross section of dimensions a x a. Let the z axis be the axis of the wave guide. Suppose the region $z < 0$ is vacuum, and the region $z > 0$ is a dielectric with permittivity ε. Write a solution of the wave equations and boundary conditions such that there is an incident and reflected wave for $z < 0$ and a transmitted wave for $z > 0$. All three waves are TE(10) waves. Determine the transmitted power as a fraction of the incident power.

[Answer: $S_{trans}/S_{inc} = 4kk'/(k+k')^2$, where k is the wave vector in the region $z < 0$ and k' is the wave vector in the region $z > 0$.]

Comments/hints: If you set it up right you can make it look pretty darn similar to the normal incidence reflection that we've done in the past and have very little actual work left to do. Just something to consider. But do note that the I expression for plane waves, $I = \frac{1}{2}\varepsilon v E^2$, isn't guaranteed to be valid in this situation.

b) When we found reflection and transmission coefficients in the free-space normal incidence situation, were R & T functions of frequency? That is, did different colors reflect differently? What about now, doing the same thing in a waveguide?