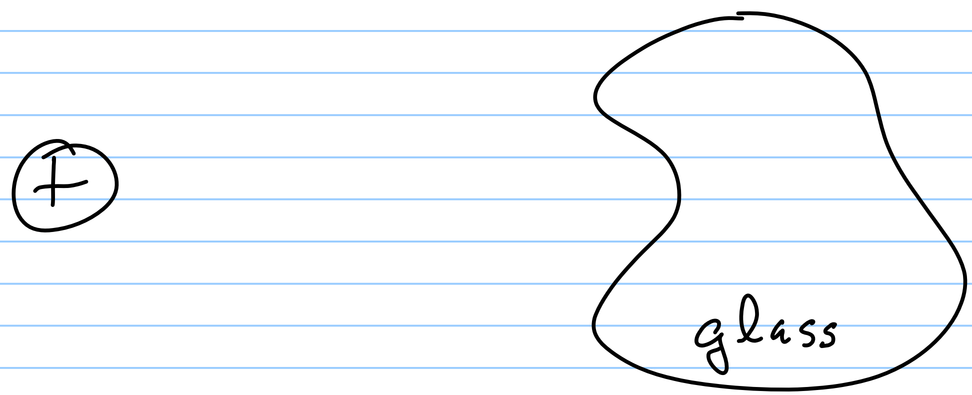
Note Title 2/26/2010

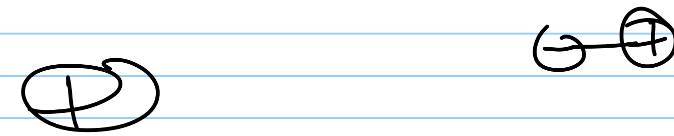
Chapter 3: hmwk due next Wednesday

What happens to $\vec{E}$ when we have matter (rather than a conductor) near a charge?

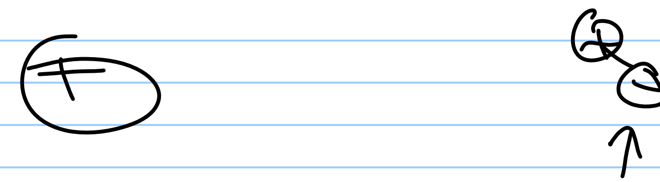


2 dipole types

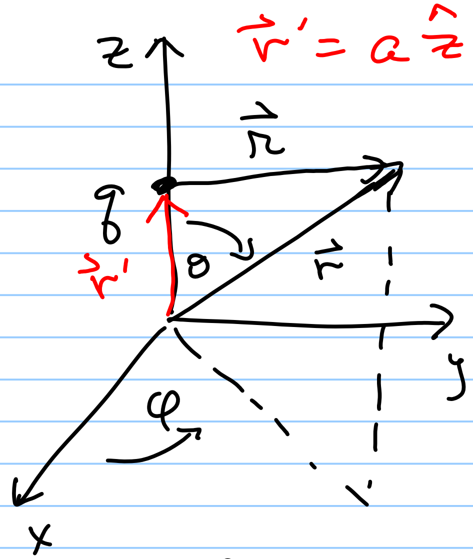
- induced



- permanent



permanent dipoles



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{q}{4\pi\epsilon_0}\frac{1}{\left(r^2+a^2-2ar\cos\theta\right)^{1/2}}$$

for $r > a$ expand in small parameter ϵ

$$r^2+a^2-2ar\cos\theta = r^2\Big(1+\underbrace{\frac{a^2}{r^2}-\frac{2a}{r}\cos\theta}_{\epsilon}\Big)$$

for all φ

$$V(r,\theta) = V(x,y,z) = \frac{q}{4\pi\epsilon_0}\frac{1}{r(1+\epsilon)^{1/2}}$$

not primed variables!

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{r}\left[1-\frac{1}{2}\left(\frac{a^2}{r^2}-\frac{2a}{r}\cos\theta\right)+\frac{3}{8}\left(\frac{a^2}{r^2}-\frac{2a}{r}\cos\theta\right)^2+\cdots\right]$$

Now arrange terms in powers of $\frac{a}{r} \ll 1$

$$\frac{1}{r}\frac{1}{(1+\epsilon)^{1/2}} = \frac{1}{r}\Big[1 + \Big(\frac{a}{r}\Big)\underbrace{\cos\theta}_{P_1(\cos\theta)} + \Big(\frac{a}{r}\Big)^2 \underbrace{\Big\{\frac{3\cos^2\theta - 1}{2}\Big\}}_{P_2(\cos\theta)} + \cdots$$

$$= \frac{1}{r}\sum_{\ell=0}^{\infty}\Big(\frac{a}{r}\Big)^{\ell} P_\ell(\cos\theta)$$

Check: 1.) for $a \ll r$ V is due to point charge $\frac{1}{4\pi\epsilon_0}\frac{q}{r}$

even though q is not at $r=0$

not r!

2.) for $a < r$ V is a complicated series expansion

AXIAL CHARGE DISTRIBUTIONS

Use superposition principle for an arbitrary charge distribution on the z axis.

$$\int_{-\infty}^{\infty} \delta(x')\,dx' = 1$$

$$\rho(\vec{r}') = \delta(x')\,\delta(y')\,\lambda(z')$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')\,d\tau'}{r} = \frac{1}{4\pi\epsilon_0}\iiint \delta(x')\,\delta(y')\,\frac{\lambda(z')\,dz'}{\left(r^2 + z'^2 - 2z'r\cos\theta\right)^{1/2}}\,dx'\,dy'$$

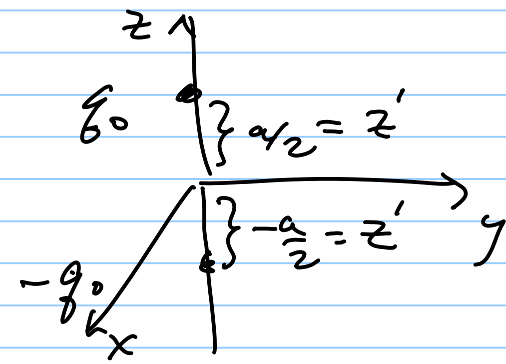
$$= \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(z')}{r}\sum_{l=0}^{\infty}\left(\frac{z'}{r}\right)^l P_l(\cos\theta)\,dz' = \frac{1}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{M_l\,P_l(\cos\theta)}{r^{l+1}}$$

where $M_l = \int \lambda(z')\,(z')^l\,dz'$ are axial multipole moments

$M_0 = \int \lambda(z')\,dz'$ is the monopole moment $\quad V_0 \propto \frac{1}{r}$

$M_1 = \int \lambda(z')\, z'\,dz'$ is the $\quad V_0 \propto \frac{1}{r^2}$

Example:



$$\lambda(z') = q_0\,\delta\left(z' - \frac{a}{2}\right) - q_0\,\delta\left(z' + \frac{a}{2}\right)$$

Dipole

$$M_1 = \int q_0\,\delta\left(z' - \frac{a}{2}\right) z'\,dz' - \int q_0\,\delta\left(z' + \frac{a}{2}\right) z'\,dz'$$

blows up a $z' = a/2$ $\qquad$ blows up at $z' = -a/2$

$$M_1 = q_0 \frac{a}{2} - q_0\left(-\frac{a}{2}\right) = q_0 a$$

Qudrapole is $l = 2$

$$M_l = \int \lambda(z')(z')^l \, dz' \quad \text{are axial multipole moments}$$

$$M_2 = \int q_0 \, \delta\left(z' - \frac{a}{2}\right)(z')^2 dz' + \int (-q_0)\, \delta\left(z' + \frac{a}{2}\right)(z')^2 dz'$$

blows up at $z' = \frac{a}{2} \Rightarrow$ put

blows up at $z' = -\frac{a}{2}$ put

$$M_2 = q_0 \left(\frac{a}{2}\right)^2 - q_0\left(-\frac{a}{2}\right)^2 = 0$$