10-24-07

Note Title 10/24/2007

Fourier integral

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/l}$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$c_n = \frac{1}{2l}\int_{-l}^{l} f(x)\, e^{-\frac{in\pi x}{l}}\,dx$$

look at c_n terms

as n increases frequency of the fourier basis fcns increase.

$$\cos\left(\frac{n\pi x}{l}\right) = \cos\left(\frac{n}{2l}2\pi x\right)$$

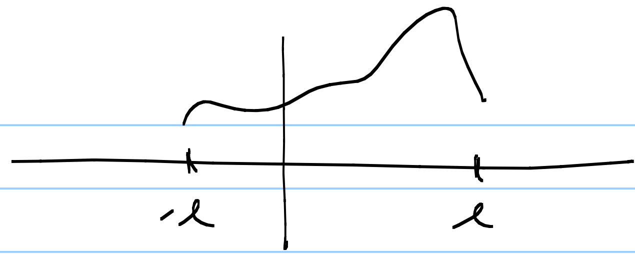
$n=1 \quad \frac{1}{2l} = f_1$

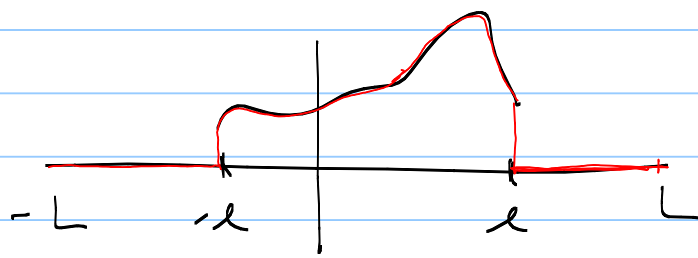
$n=2 \quad \frac{1}{l} = f_2$

$n=3 \quad \frac{3}{2l} = f_3$

$\vdots$

$\frac{k}{2l} = f_k$





let $2\ell = L$

Now the frequencies are

$$\frac{1}{2L} = \frac{1}{4\ell}$$

$$\frac{2}{2L} = \frac{1}{2\ell} \rightarrow \text{old } f_1$$

$$\frac{3}{2L} = \frac{3}{4\ell}$$

$$\frac{4}{2L} = \frac{1}{\ell} \rightarrow \text{old } f_2$$

If I keep repeating this process, frequency spacing gets smaller and smaller

$$f(x) = \sum_{N=-\infty}^{\infty} c_N \, e^{i \frac{N\pi}{L} x}$$

$$\Downarrow$$

$$\propto \int_{-\infty}^{\infty} F(k) \, e^{ikx} \, dk$$

in fact

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) \, e^{ikx} \, dk$$

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \, e^{-ikx} \, dx$$

The latter is the "Fourier transform" of $f(x)$. These 2 equations imply the invertibility of the transform.

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(x') \, e^{-ikx'} \, dx' \right] e^{ikx} \, dk$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x') \underbrace{\left[\int_{-\infty}^{\infty} e^{-ik(x'-x)} \, dk \right]}_{=?} dx$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ik(x'-x)}\, dk$$

$$= \lim_{\mu \to \infty} \underbrace{\frac{1}{2\pi}\int_{-\mu}^{\mu} e^{-ik(x'-x)}\, dk}_{\frac{\sin \mu (x'-x)}{\pi (x'-x)}}$$

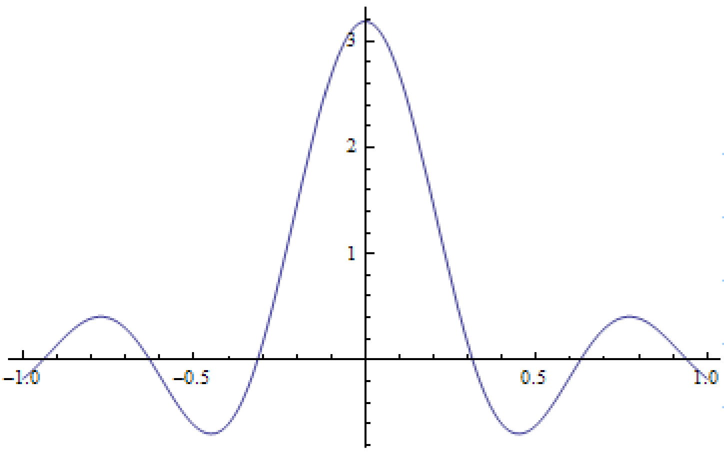
what is this limit

$$\lim_{\mu \to \infty} \frac{\sin \mu (x'-x)}{\pi (x'-x)}$$

Lets plot it

$$\frac{\sin \mu x}{\pi x}$$

Out[5]=



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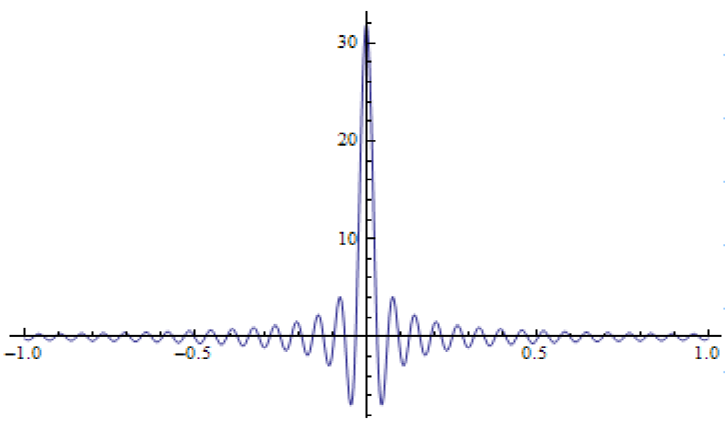

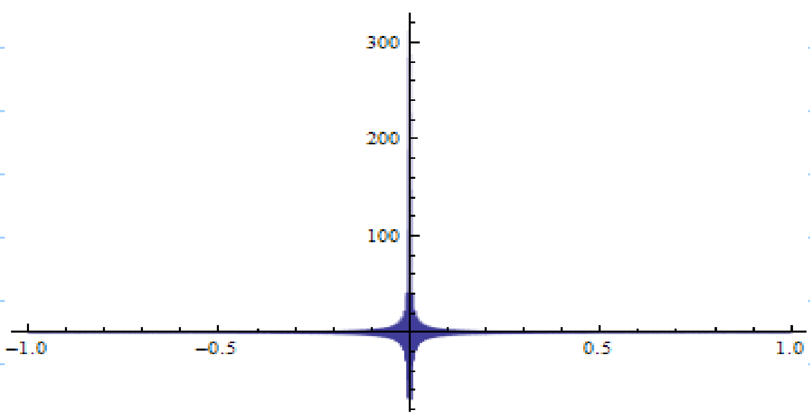

converges to $\delta(x)$

so $$f(x) = \int_{-\infty}^{\infty} f(x')\,\delta(x'-x)\,dx$$