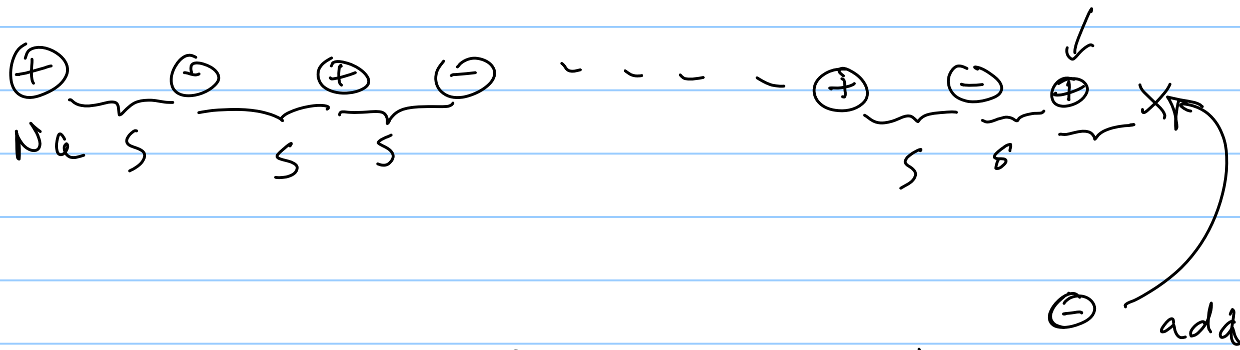
Hmwk due next Friday Jan 25

Ch 1: 46, 53

Ch 2: 30, 31, 32, 33, 35, 39, 43, 51

Energy

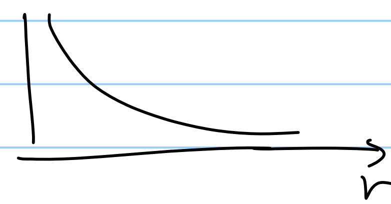
Salt crystal 1-D: dissociation energy



$$W_{me} = \Delta PE = PE_f - PE_i = q(V_f - V_i)$$

$V_i = 0$ since ⊕ is at ∞

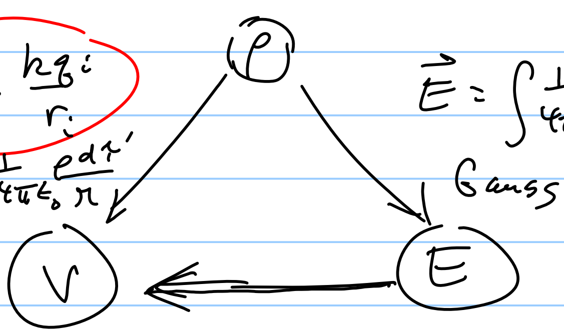
$$V_f = \frac{ke}{s} - \frac{ke}{2s} + \frac{ke}{3s} - \frac{ke}{4s} + \cdots$$



$$V = \sum_i \frac{kq_i}{r_i}$$

$$\to \int \frac{1}{4\pi\epsilon_0} \frac{\rho\, d\tau'}{r}$$

$$\vec{E} = \int \frac{1}{4\pi\epsilon_0} \frac{\overbrace{\rho\, d\tau'}^{dq}}{r^2} \hat{r}$$



$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

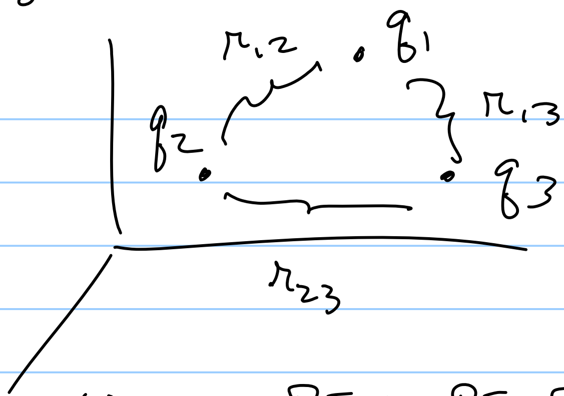
$$-1 < x \le 1$$

$$\ln[1+x] = 1 - \frac{x}{2} + \frac{x^2}{2} - \cdots$$

$$V_f = \frac{e}{4\pi\epsilon_0} \underbrace{\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots \right]}_{\ln 2 = .693}$$

$$\Delta PE = PE_f = q_1 V_f = -\frac{e^2}{4\pi\epsilon_0}(.693)$$

Bring together 3 charges



$$W_{me} = \Delta PE = PE_f - PE_i$$

($PE_i \to 0$ since 3 charge are initially at ∞)

Bring in 1st charge $W_{me} = 0$

" " 2nd " $\quad W = \dfrac{k q_1 q_2}{r_{12}}$

Bring in 3rd " $\quad W = \dfrac{k q_1 q_2}{r_{12}} + \dfrac{k q_3 q_1}{r_{13}} + \dfrac{k q_3 q_2}{r_{23}}$

$$W = \frac{1}{2}\left\{ \frac{q_1}{4\pi\epsilon_0}\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}}\right) + \frac{q_2}{4\pi\epsilon_0}\left(\frac{q_1}{r_{12}} + \frac{q_3}{r_{23}}\right) + \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)\right\}$$

Voltage at point 1 due to q_2 & q_3 — voltage at point 2 due to q_1 & q_3 — voltage at point 3 due to q_1 & q_2

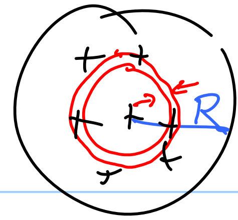
$$W = \frac{1}{2}\sum_{i=1}^{3} q_i V_i$$

If we have a continuous charge dist.

$$W = \frac{1}{2}\int dq\, V$$

(V ↑ due to all other charges present in distribution)

Ex: spherical charge distribution



$\rho = \text{const.}$

$dq = \rho\, d\tau = \rho\, 4\pi r^2 dr$

Need E

E_{out} $\qquad \Phi_E = \frac{Q_{encl}}{\epsilon_0}$

$$\oint |\vec{E}||d\vec{a}| = E\underbrace{\oint da}_{4\pi r^2} = \frac{\int \rho\, d\tau}{\epsilon_0} = \frac{\rho \frac{4}{3}\pi R^3}{\epsilon_0}$$

$$E_{out} = \frac{\rho}{3\epsilon_0}\frac{R^3}{r^2}$$

E_{inside} use spherical shell inside

$$E\, 4\pi r^2 = \frac{\int \rho\, d\tau}{\epsilon_0} = \frac{\rho}{\epsilon_0}\frac{4}{3}\pi r^3$$

$$E = \frac{\rho r}{3\epsilon_0}$$

$$\Delta V = -\int E \cdot dl = -\int_{\infty}^{R} E_{out}\, dr - \int_{R}^{r} E_{in}\, dr$$

$V_f - V_i$ (V_i = 0 at ∞)

$$V(r) = \frac{3}{6}\frac{\rho R^2}{\epsilon_0} - \frac{\rho r^2}{6\epsilon_0}$$

$$W_{me} = \frac{1}{2}\int_0^R \underbrace{\frac{\rho}{6\epsilon_0}(3R^2 - r^2)}_{V}\, \underbrace{\rho\, d\tau}_{\rho 4\pi r^2 dr} = \frac{4\pi\rho^2 R^5}{15\,\epsilon_0}$$

Ex: another way is to assemble the charge a shell at a time

$dW = dq\,V$

$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$, $\quad Q = \rho\frac{4\pi r^3}{3}$

$$W = \int V\,dq = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{\rho\frac{4}{3}\pi r^3}{r}\,\rho\,4\pi r^2\,dr = \frac{4}{15}\frac{\pi\rho^2}{\epsilon_0}R^5$$