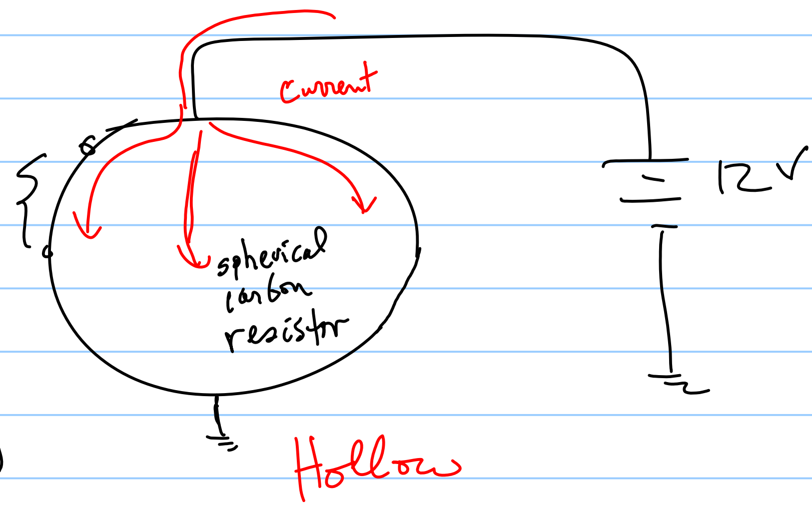
Note Title 2/11/2009

$$V(r,\theta) = \sum_{\ell=0}^{\infty} \left(A_\ell r^\ell + B_\ell r^{-(\ell+1)}\right) P_\ell(\cos\theta)$$

$V(\theta)$ given by

$\Delta V = IR$



Find $V_{in}(r,\theta)$ & $V_{out}(r,\theta)$

What is the form (series) for $V(r,\theta)$ inside?

$$V_{in}(r,\theta) = \sum_{\ell=0}^{\infty} A_\ell r^\ell P_\ell(\cos\theta)$$

ODE solns.

$$\frac{d^2 \Psi(x)}{dx^2} = C\,\Psi(x)$$

assume series soln $\Psi(x) = \sum_n a_n x^n$

looks like a Taylor expansion around $x_0 = 0$

LHS $\sum_n a_n n(n-1) x^{n-2} \propto \sum_n a_n x^n$

set coeff of x^n on LHS = RHS

$$\int_{-1}^{1} P_\ell(x)\, P_m(x)\, dx = \int_0^{\pi} P_\ell(\cos\theta) P_m(\cos\theta) \sin\theta\, d\theta = \frac{2}{2m+1}\delta_{m\ell}$$

$$V_{in}(R,\theta) = \sum_{\ell=0}^{\infty} A_\ell R^\ell P_\ell(\cos\theta) = V(\theta)$$

find A_ℓ ↑ ↑ given

multiply both sides by $P_m(\cos\theta)\, d(\cos\theta)$ & integrate

$$\int_{-1}^{1} V(\theta) P_m(\underbrace{\cos\theta}_{x}) d\theta = \sum_{\ell=0}^{\infty} A_\ell R^\ell \overbrace{\int_{-1}^{1} P_\ell(\cos\theta) P_m(\cos\theta)\, \underbrace{d(\cos\theta)}_{dx}}^{\frac{2}{2m+1}\delta_{\ell m}}$$

$$= A_m R^m \frac{2}{2m+1}$$

$$\boxed{A_m^{\text{inside}} = \frac{(2m+1)}{2R^m} \int_{-1}^{1} V(\theta) P_m(\cos\theta)\, d(\cos\theta)}$$

$m \leftrightarrow \ell$

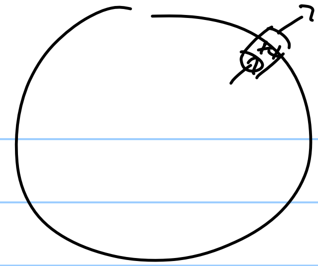
We know A_ℓ & do similar outside to get B_ℓ

$$\underbrace{V(r,\theta)}_{\text{all space}} = \sum_{\ell=0}^{\infty} \Big(\underbrace{A_\ell r^\ell}_{\text{ONLY INSIDE}} + \underbrace{B_\ell r^{-(\ell+1)}}_{\text{ONLY OUTSIDE}} \Big) P_\ell(\cos\theta)$$

find $\vec{E} = -\vec{\nabla} V$

↑ spherical coords

find σ on sphere



Ex: $V(\theta)$