Conventions in optics

- several ways to write an expression for waves that are physically the same.
- conventions are used to be consistent and avoid mistakes.

$\underline{e^{-i\omega t}}$

physical field is $\mathrm{Re}\left(e^{\pm i(kz-\omega t)}\right) = \cos(kz-\omega t)$

- relative sign btw kz, ωt determines wave direction
- overall sign of $(kz-\omega t)$ is not physically important <u>when you take the real part</u>

$\rightarrow$ we choose $+$ so that the time dependence is $e^{-i\omega t}$

This is critically important when we introduce complex ref. index

<u>circular polarization</u>

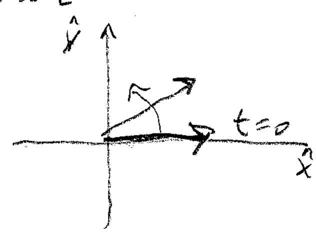
which way does $\vec{E}$ spin?

for state $\begin{pmatrix} 1 \\ i \end{pmatrix}$ write $\vec{E}(t) = (\hat{x} + i\hat{y})e^{-i\omega t}$ at $z=0$

real field is $\mathrm{Re}(\vec{E}) = \hat{x}\cos\omega t + \hat{y}\sin\omega t$

at $t=0$ $\vec{E} \sim \hat{x}$

for $t \gtrsim 0$ E_x decreases E_y increases $\rightarrow$



$\therefore$ rotation is CCW

note $\hat{k} = \hat{z}$ (out of page)

Normal right hand rule would call this RH

<u>but</u> convention (from 1800's) is opposite.

$\therefore$ $\begin{pmatrix} 1 \\ i \end{pmatrix} = \mathrm{LH}$ $\begin{pmatrix} 1 \\ -i \end{pmatrix} = \mathrm{RH}$

Note $\begin{pmatrix} i \\ 1 \end{pmatrix} = i\begin{pmatrix} 1 \\ -i \end{pmatrix} \rightarrow \mathrm{RH}$

For arbitrary state, make x-compon $\rightarrow 1$ to see which way vector turns

Normalization of polarization state.
(not always done)

full field $\vec{E}(z,t) = E_0 \frac{(\hat{x} + i\hat{y})}{\sqrt{2}} e^{i(kz - \omega t + \phi_0)}$

> normalizing polariz. vector allows quantity E_0 to represent actual field strength.

> factor out absolute phase ϕ_0 to put vector into a readable form.

Propagation phase

general wave $\vec{E}(z,t) = E_0 \hat{x} e^{i(kz-\omega t)}$

$k = k_0 n$ in medium

we want to know how wave changes with propagation.

at $z=0$ $\vec{E}(0,t) = E_0 \hat{x} e^{-i\omega t}$

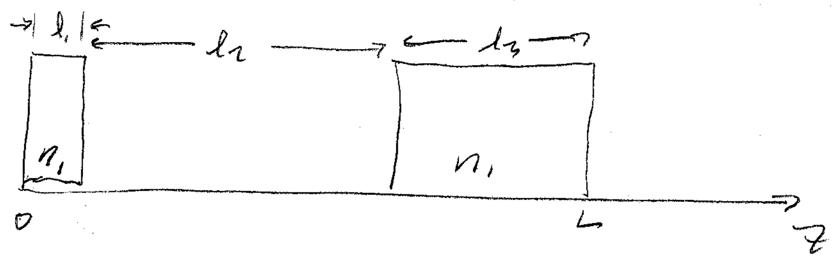
at $z=l$ $\vec{E}(l,t) = E \hat{x} e^{-i\omega t} e^{ik_0 n l}$

wave picks up a phase shift of $\phi = k_0 n l$

$nl \equiv$ "optical path"

Note that the wave is continuous (CW): it doesn't matter what time t we pick.

Example:



wave at $z = L = l_1 + l_2 + l_3$

$$\vec{E}(L,t) = E_0 \hat{x} e^{ik_0 n l_1} e^{ik_0 l_2} e^{ik_0 n l_3} e^{-i\omega t}$$
$$= E_0 \hat{x} e^{ik_0(n(l_1+l_3)+l_2)} e^{-i\omega t}$$

- we're just carrying $e^{-i\omega t}$ around, often suppress this
- wave for general $z > L$:

$\vec{E}(L,t) \cdot e^{ik(z-L)}$

Introduction to birefringence

→ index of refraction $n = \sqrt{\varepsilon}$ varies with direction

→ $\overleftrightarrow{\varepsilon}$

isotropic: $\vec{D} = \varepsilon \vec{E}$ $\qquad \varepsilon$ constant, scalar

anisotropic: $\vec{D} = \overleftrightarrow{\varepsilon} \cdot \vec{E}$ $\qquad \varepsilon$ tensor, 3x3 matrix

- general case:

  $\vec{D}$ not parallel to $\vec{E}$

- coordinates can be chosen to make $\overleftrightarrow{\varepsilon}$ diagonal:

$$\rightarrow \quad \vec{D} = \begin{pmatrix} \varepsilon_x & & \\ & \varepsilon_y & \\ & & \varepsilon_z \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

so if $\vec{E}$ is resolved along these crystal axes,

E_x "sees" $n_x = \sqrt{\varepsilon_x}$,

$E_y \rightarrow n_y$ etc.

Uniaxial: $\varepsilon_y = \varepsilon_z \rightarrow n_o^2$ "ordinary" $\quad n_e = \varepsilon_x^{1/2}$ "extraordinary"

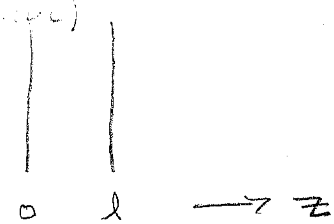
Example:

$$\vec{E} = E_0 \begin{pmatrix} a \\ b \end{pmatrix} e^{i(k_0 z - \omega t)} \quad \text{in vacuum.}$$

if crystal is aligned with axes along x, y

pass thru crystal:

$$\vec{E}(l) = E_0 \begin{pmatrix} a e^{i k_0 n_x l} \\ b e^{i k_0 n_y l} \end{pmatrix} e^{-i\omega t}$$



what is polarization state? look at <u>phase</u> <u>difference</u>

$$\begin{pmatrix} a \\ b e^{i k_0 (n_y - n_x) l} \end{pmatrix}$$

Now we have control over polarization state.
look at phase difference

$$\Delta\phi = k_0 (n_y - n_x) l$$

full waveplate "λ"

$\Delta\phi = 2\pi$ (or multiple)

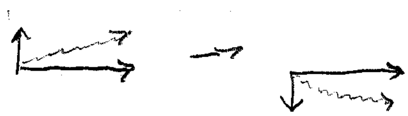
$e^{i\Delta\phi} = 1 \rightarrow$ no change

half wave plate "$\lambda/2$"

$\Delta\phi = \pi$ (or odd multiple)

$e^{i\Delta\phi} = -1$

$\begin{pmatrix} a \\ b \end{pmatrix} \rightarrow \begin{pmatrix} a \\ -b \end{pmatrix}$ still linear



polarization is rotated. by 2θ

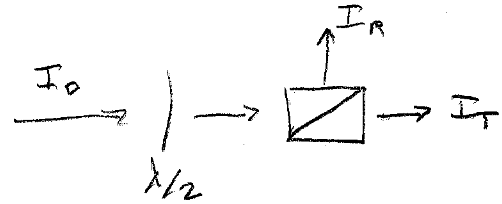
special case: $a = b$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ rotation by 90°

$45^\circ \rightarrow -45^\circ$

Applications:

- variable attenuator:



- Pockels cell: high voltage on crystal $\rightarrow$ waveplate $\rightarrow$ fast (ns) optical switch.

quarter wave plate "$\lambda/4$"

$\Delta\phi = \pi/2$ or odd multiples

if linear and 45° input

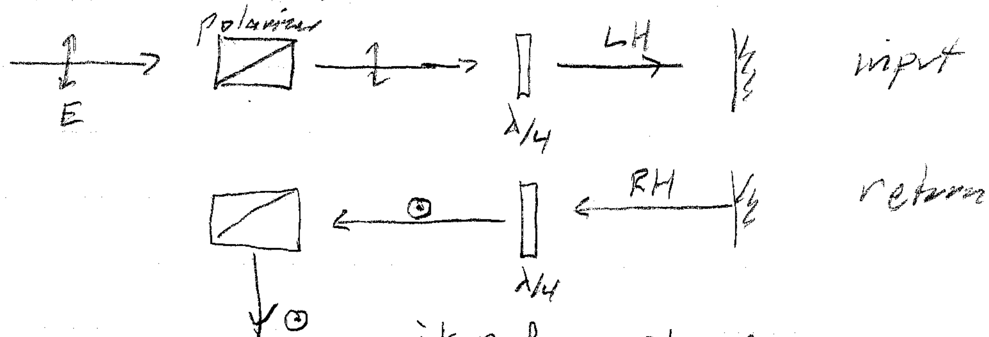
$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ e^{i\pi/2} \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix}$$ circular (Left)

or, rotate $\lambda/4$ plate by 90°

then $n_y - n_x \rightarrow n_x - n_y$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ e^{-i\pi/2} \end{pmatrix} = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$ RH circ.

Application: isolator



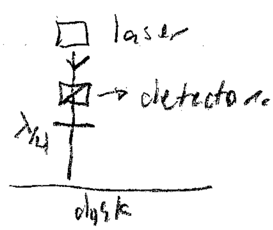
E_x gets phase $e^{ik_0 n_x l} \cdot e^{ik_0 n_x l}$ $\rightarrow \phi_x = 2k_0 n_x l$

E_y $\rightarrow \phi_y = 2k_0 n_y l$

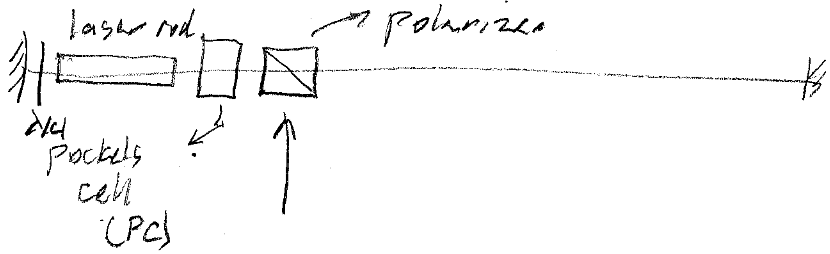
i.e. if $\Delta\phi = \pi/2$ on one pass

$\Delta\phi = \pi$ on 2

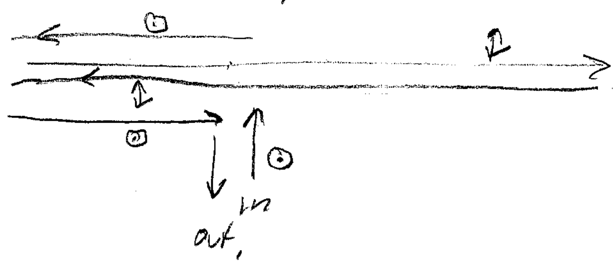
- prevent feedback into laser. (or damage)
- CD read out



"regenerative" amplifier for short pulses (< 1 ns)



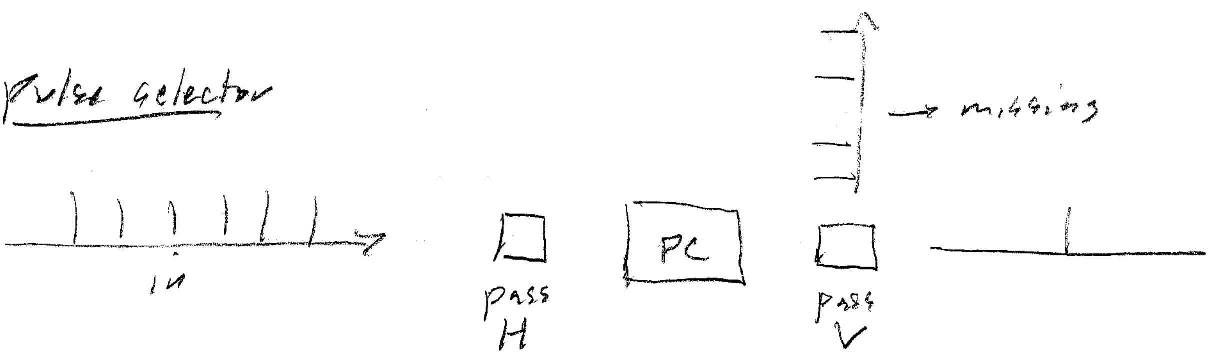
PC off → 4 passes



turn PC on to $\lambda/4$ voltage after 1st double pass
pulse is trapped inside

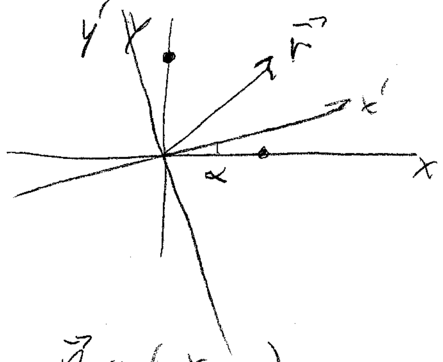
turn PC off after # round trips (e.g. 20)
dump pulse out.

Pulse selector



PC off : no light there
PC on to $\lambda/2$ → pass pulse
PC off : reject pulses

Coordinate transformations with matrices



$\vec{r} = (x, y)$

$\vec{r}' = (x', y')$

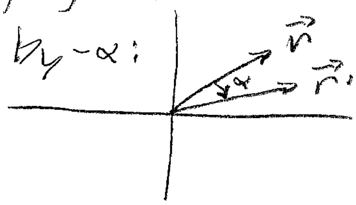
$x' = x\cos\alpha + y\sin\alpha$

$y' = -x\sin\alpha + y\cos\alpha$

write as

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \vec{r}' = R_\alpha \cdot \vec{r}$$

here we are rotating the coordinate system, not the vector,
we can also imagine keeping the coordinate system fixed,
and rotating the vector by $-\alpha$:



Properties of rotation matrices:

inverse: $R_\phi^{-1} = R_{-\phi}$

$R_{-\phi} R_\phi = I$ $\quad \begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} c & s \\ -s & c \end{pmatrix} = \begin{pmatrix} c^2+s^2 & 0 \\ 0 & c^2+s^2 \end{pmatrix} = I$

eigenvalues:

eqn $\quad M\vec{r}_\lambda = \lambda \vec{r}_\lambda \qquad \lambda =$ eigenvalue

$\vec{r}_\lambda =$ eigenvector

find λ:

$$R_\phi \vec{r}_\lambda - I\vec{r}_\lambda = 0 \rightarrow \begin{bmatrix} \cos\phi - \lambda & \sin\phi \\ -\sin\phi & \cos\phi - \lambda \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$$

$\therefore \det(R_\phi - I\lambda) = 0$

$$(\cos\phi - \lambda)^2 + \sin^2\phi = 0$$

$$\cos^2\phi - 2\lambda\cos\phi + \lambda^2 + \sin^2\phi = 0$$

→ quadratic eqn for 2 roots:

$$\lambda^2 - 2\cos\phi\,\lambda + 1 = 0$$

$$\lambda = \frac{-(-2\cos\phi) \pm \sqrt{4\cos^2\phi - 4}}{2} = \cos\phi \pm \sqrt{-\sin^2\phi}$$

$$= e^{\pm i\phi}$$

eigenvectors: $(\cos\phi - e^{\pm i\phi})\,a + (\sin\phi)\,b = 0$

$$\rightarrow b = \frac{\cos\phi - e^{\pm i\phi}}{\sin\phi}\,a$$

$$= \frac{\cos\phi - \cos\phi \mp i\sin\phi}{\sin\phi} = \mp i\,a$$

normalize $\vec{r}_\lambda^{\,+}\vec{r}_\lambda = 1$

$$|c|^2 (1 \quad \pm i)\begin{pmatrix} 1 \\ \mp i \end{pmatrix} = 2$$

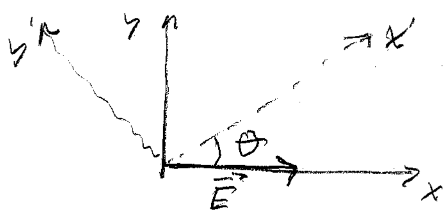
eigenvectors are: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ for $\lambda = e^{-i\phi}$

$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$ $\lambda = e^{+i\phi}$

Note that these correspond to RH, LH circ. polariz! these states are the fundamental polarization states of photons $\pm\hbar$ spin angular momentum.

Arbitrary angle w.r.t ~~a~~ crystal axes:

$E_x = E_0$
$E_y = 0$



crystal axes are rotated by θ

$E_{x'} = E_0 \cos\theta$
$E_{y'} = -E_0 \sin\theta$

$$\rightarrow E_0 \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}$$

now can apply crystal phase:

$$E_{out} = E_0 \begin{pmatrix} \cos\theta \, e^{i\phi_x} \\ -\sin\theta \, e^{i\phi_y} \end{pmatrix}$$

Alternate method: use rotation matrix to change basis

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}$$

Waveplate of arbitrary orientation

express input wave in terms of crystal axes,

$$\hat{\varepsilon}_{in} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \hat{\varepsilon}'_{in} = \overleftrightarrow{R}_{\phi} \cdot \hat{\varepsilon}_{in}$$

$$= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sin\phi \\ \cos\phi \end{pmatrix}$$

$$\hat{\varepsilon}'_{out} = \overleftrightarrow{W} \cdot \hat{\varepsilon}'_{in}$$

$$= \begin{pmatrix} e^{ik_o n_e d} & 0 \\ 0 & e^{ik_o n_o d} \end{pmatrix} \begin{pmatrix} \sin\phi \\ \cos\phi \end{pmatrix}$$

$$= e^{ik_o n_e d} \begin{pmatrix} \sin\phi \\ e^{ik_o d(n_e - n_o)} \cos\phi \end{pmatrix}$$

if we want, we can express this in the original basis

$$\hat{\varepsilon}_{out} = R_{-\phi} \cdot \hat{\varepsilon}'_{out}$$

$$= \underbrace{R_{-\phi}\, W\, R_{\phi}}_{R^{-1} W R \,=\, W' \text{ rotated matrix.}} \hat{\varepsilon}_{in}$$