Problem 7.33

(a) $\mathbf{J}_d = \epsilon_0 \frac{\mu_0 I_0 \omega^2}{2\pi} \cos(\omega t) \ln(a/s)\,\hat{\mathbf{z}}$. But $I_0 \cos(\omega t) = I$. So $\boxed{\mathbf{J}_d = \frac{\mu_0 \epsilon_0}{2\pi} \omega^2 I \ln(a/s)\,\hat{\mathbf{z}}.}$

(b) $I_d = \int \mathbf{J}_d \cdot d\mathbf{a} = \frac{\mu_0 \epsilon_0 \omega^2 I}{2\pi} \int_0^a \ln(a/s)(2\pi s\, ds) = \mu_0 \epsilon_0 \omega^2 I \int_0^a (s \ln a - s \ln s) ds$

$= \mu_0 \epsilon_0 \omega^2 I \left[(\ln a) \frac{s^2}{2} - \frac{s^2}{2} \ln s + \frac{s^2}{4} \right] \Big|_0^a = \mu_0 \epsilon_0 \omega^2 I \left[\frac{a^2}{2} \ln a - \frac{a^2}{2} \ln a + \frac{a^2}{4} \right] = \boxed{\frac{\mu_0 \epsilon_0 \omega^2 I a^2}{4}.}$

Problem 7.37

$E = \frac{V}{d} \Rightarrow J_c = \sigma E = \frac{1}{\rho} E = \frac{V}{\rho d}$. $J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\epsilon E) = \epsilon \frac{\partial}{\partial t} \left[\frac{V_0 \cos(2\pi \nu t)}{d} \right] = \frac{\epsilon V_0}{d} [-2\pi\nu \sin(2\pi\nu t)]$.

The ratio of the amplitudes is therefore:

$$\frac{J_c}{J_d} = \frac{V_0}{\rho d} \frac{d}{2\pi\nu\epsilon V_0} = \frac{1}{2\pi\nu\epsilon\rho} = \left[2\pi (4 \times 10^8)(81)(8.85 \times 10^{-12})(0.23) \right]^{-1} = \boxed{2.41.}$$

Problem 7.53

Let Φ be the flux of **B** through a *single* loop of either coil, so that $\Phi_1 = N_1 \Phi$ and $\Phi_2 = N_2 \Phi$. Then

$$\mathcal{E}_1 = -N_1 \frac{d\Phi}{dt}, \quad \mathcal{E}_2 = -N_2 \frac{d\Phi}{dt}, \text{ so } \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}. \quad \text{qed}$$

Problem 7.54

(a) Suppose current I_1 flows in coil 1, and I_2 in coil 2. Then (if Φ is the flux through *one* turn):

$$\Phi_1 = I_1 L_1 + M I_2 = N_1 \Phi; \quad \Phi_2 = I_2 L_2 + M I_1 = N_2 \Phi, \quad \text{or } \Phi = I_1 \frac{L_1}{N_1} + I_2 \frac{M}{N_1} = I_2 \frac{L_2}{N_2} + I_1 \frac{M}{N_2}.$$

In case $I_1 = 0$, we have $\frac{M}{N_1} = \frac{L_2}{N_2}$; if $I_2 = 0$, we have $\frac{L_1}{N_1} = \frac{M}{N_2}$. Dividing: $\frac{M}{L_1} = \frac{L_2}{M}$, or $L_1 L_2 = M^2$. qed

(b) $-\mathcal{E}_1 = \frac{d\Phi_1}{dt} = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} = V_1 \cos(\omega t)$; $-\mathcal{E}_2 = \frac{d\Phi_2}{dt} = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} = -I_2 R$. qed

(c) Multiply the first equation by L_2: $L_1 L_2 \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} M = L_2 V_1 \cos\omega t$. Plug in $L_2 \frac{dI_2}{dt} = -I_2 R - M \frac{dI_1}{dt}$.

$M^2 \frac{dI_1}{dt} - MRI_2 - M^2 \frac{dI_1}{dt} = L_2 V_1 \cos\omega t \Rightarrow \boxed{I_2(t) = -\frac{L_2 V_1}{MR} \cos\omega t.}$ $L_1 \frac{dI_1}{dt} + M \left(\frac{L_2 V_1}{MR} \omega \sin\omega t \right) = V_1 \cos\omega t$.

$$\frac{dI_1}{dt} = \frac{V_1}{L_1} \left(\cos\omega t - \frac{L_2}{R} \omega \sin\omega t \right) \Rightarrow \boxed{I_1(t) = \frac{V_1}{L_1} \left(\frac{1}{\omega} \sin\omega t + \frac{L_2}{R} \cos\omega t \right).}$$

(d) $\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{I_2 R}{V_1 \cos\omega t} = \frac{-\frac{L_2 V_1}{MR} \cos\omega t\, R}{V_1 \cos\omega t} = -\frac{L_2}{M} = -\frac{N_2}{N_1}$. The ratio of the amplitudes is $\frac{N_2}{N_1}$. qed

(e) $P_{\text{in}} = V_{\text{in}} I_1 = (V_1 \cos\omega t)(\frac{V_1}{L_1}) \left(\frac{1}{\omega} \sin\omega t + \frac{L_2}{R} \cos\omega t \right) = \boxed{\frac{(V_1)^2}{L_1} \left(\frac{1}{\omega} \sin\omega t \cos\omega t + \frac{L_2}{R} \cos^2\omega t \right).}$

$P_{\text{out}} = V_{\text{out}} I_2 = (I_2)^2 R = \boxed{\frac{(L_2 V_1)^2}{M^2 R} \cos^2\omega t.}$ Average of $\cos^2\omega t$ is 1/2; average of $\sin\omega t \cos\omega t$ is zero.

So $\langle P_{\text{in}} \rangle = \frac{1}{2}(V_1)^2 \left(\frac{L_2}{L_1 R} \right)$; $\langle P_{\text{out}} \rangle = \frac{1}{2}(V_1)^2 \left[\frac{(L_2)^2}{M^2 R} \right] = \frac{1}{2}(V_1)^2 \left[\frac{(L_2)^2}{L_1 L_2 R} \right]$; $\boxed{\langle P_{\text{in}} \rangle = \langle P_{\text{out}} \rangle = \frac{(V_1)^2 L_2}{2 L_1 R}.}$