Note Title 2/1/2007

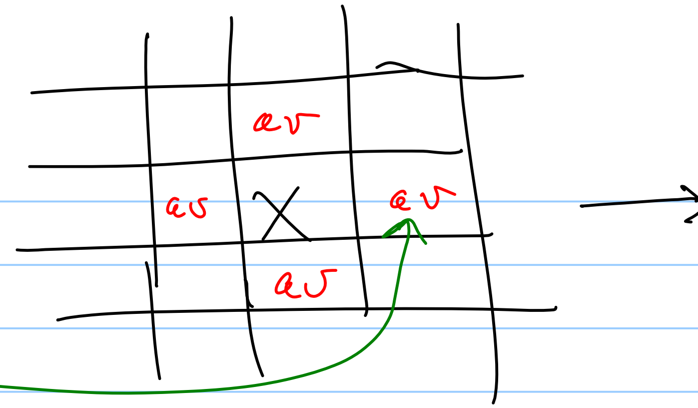
-Equation sheet must be turned in Tuesday noon: No late sheets accepted. One sheet of paper but you can write on both sides. Only formulas allowed. No examples, problems, etc.
-Ask any question you have about the exam on the forum. I will answer up to noon Tuesday.
-Two 2006 exams are on the forum. They are midterms and so won't be of much use.

On this exam, I expect you to be able to

(1) be able to calculate E using Gauss's law given a symmetric charge distribution and calculate E using Coulomb's laws given an arbitrary charge distribution.
(2) Be able to apply the differential form of Gauss's law and understand what it means.
(3) Be able to calculate V given charge distribution by the two methods in the triangle diagram.
(4) Be able to execute the Divergence and Stokes theorems in Cartesian and spherical coordinates.
(5) Be able to calculate Eperp and Eparallel across a boundary using the divergence and Stokes theorems.
(6) Find an integral expression for the energy required to assemble a charge distribution.
(7) Understand how to derive a solution to Laplace's equation using separation of variables and apply it to a simple case in Cartesian coordinates.

Drop 3.4 & 3.15 on next assign.

Relaxation method



Taylor series

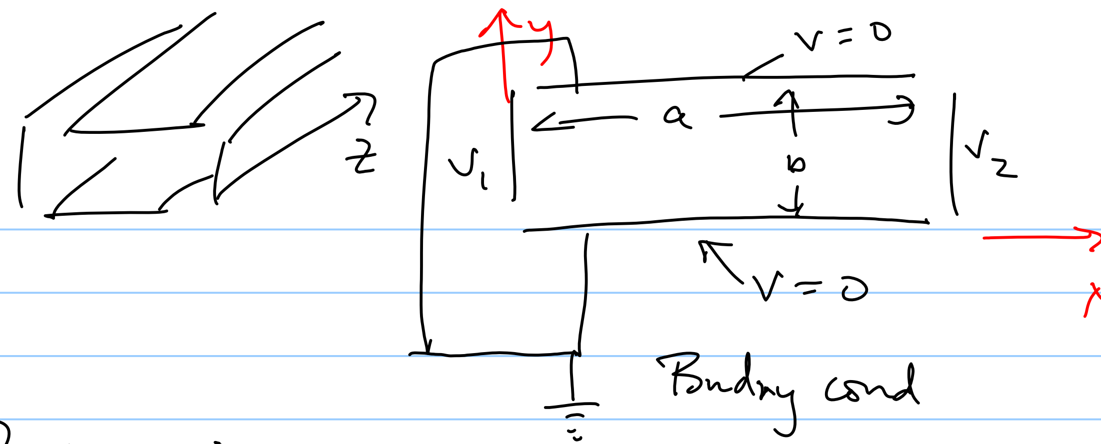
$$V(x_0+\delta, y_0, z_0) = V(x_0, y_0, z_0) + \delta \frac{\partial V}{\partial x} + \frac{\delta^2}{2}\frac{\partial^2 V}{\partial x^2} + \frac{\delta^3}{6}\frac{\partial^3 V}{\partial x^3} + \cdots$$

$$V(x_0-\delta, y_0, z_0) = V(x_0, y_0, z_0) - \delta \frac{\partial V}{\partial x} + \frac{\delta^2}{2}\frac{\partial^2 V}{\partial x^2} + \cdots$$

$$V(x_0, y_0+\delta, z_0) = V(x_0, y_0, z_0) + \delta \frac{\partial V}{\partial y} + \frac{\delta^2}{2}\frac{\partial^2 V}{\partial y^2} + \cdots$$

$$\bar{V} = \frac{1}{6}\left[6V(x_0, y_0, z_0) + \delta^2 \underbrace{\left\{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right\}}_{\nabla^2 V = 0} + \delta^4(\quad) + \cdots\right]$$

Sep. Varibles



$V(x,y,z) = X(x)\,Y(y)\,Z(z)$ 3 - ODE

$Y = A \sin ky + B \cos ky$ $\qquad c_2 < 0$

$Y = A' e^{\sqrt{c_2}\,y} + B' e^{-\sqrt{c_2}\,y}$ $\qquad c_2 > 0$

$Y = A'' + B''y$ $\qquad c_2 = 0$

$$\nabla^2 V = 0 = \underbrace{\frac{1}{X}\frac{d^2 X}{dx^2}}_{c_1} + \underbrace{\frac{1}{Y}\frac{d^2 Y}{dy^2}}_{c_2} + \underbrace{\frac{1}{Z}\frac{d^2 Z}{dz^2}}_{c_3}$$

$$0 = c_1 + c_2 + c_3$$

What is C_3? Field & potential in z direction indep of z.

$Z(z) = $ constant $\qquad C_3 = 0$

The voltage at boundaries y=0 and y=b is zero. Derive an expression for Y (y).

$Y(y) = A \sin ky + B \cos ky$

$Y(0) = 0 = 0 + B\cos(0) = B \Rightarrow B = 0$

$Y(b) = 0 = A \sin kb \qquad kb = n\pi, \quad n = 1, 2, 3, \dots$

$$\boxed{k = \frac{n\pi}{b}}$$

$$Y(y) = A \sin\left(\frac{n\pi}{b} y\right)$$

$\Rightarrow C_2 = -k^2 = -\left(\frac{n\pi}{b}\right)^2$

$$C_1 + C_2 = 0 \qquad C_1 = k^2 = \left(\frac{n\pi}{b}\right)^2 > 0$$

$$C_1 - {}''k^2 = 0$$

$$\frac{d^2 X}{dx^2} = k^2 X \qquad X = G e^{kx} + H e^{-kx}$$

$$V(x,y,z) = X(x)\,Y(y)\,\underbrace{Z(z)}_{\text{const}} = \left(G e^{kx} + H e^{-kx}\right) A \sin\left(\frac{n\pi}{b} y\right) L$$

$$= \left(G e^{kx} + H e^{-kx}\right) \sin ky$$