PHGN 480: Laser Physics due 23Sept 2011
Homework 4

1) Svelto problem 2.12

2) A more general derivation of the saturation in a two level system. In this version, we have pump rates (/vol/time) R_1 and R_2, and overall lifetimes out of the levels τ_1 and τ_2. We also include the effect of degeneracies on levels 1 and 2, g_1 and g_2. We'll work with the beam intensity and the cross-section rather than the pump rate $W = \frac{I\sigma_{21}}{h\nu_L}$. With these included, the rate equations are

$$\frac{dN_2}{dt} = R_2 - \Delta N^* \sigma_{21} \frac{I}{h\nu_L} - \frac{N_2}{\tau_2}$$
$$\frac{dN_1}{dt} = R_1 + \Delta N^* \sigma_{21} \frac{I}{h\nu_L} - \frac{N_1}{\tau_1} + N_2 A_{21}$$

where

$$\Delta N^* = N_2 - \frac{g_2}{g_1} N_1$$

is the population inversion density. Remember that in this formulation, spontaneous emission out of level 2 is included in the lifetime in that level, τ_2.

a. First set the time derivatives = 0 for steady state, then calculate the steady state expressions for N_1^{ss} and N_2^{ss}. Then calculate the steady-state value of ΔN^*, to show that

$$\Delta N^*(I) = N_2^{ss} - \frac{g_2}{g_1} N_1^{ss} = \frac{R_2\tau_2\left[1-(g_2/g_1)A_{21}\tau_1\right] - (g_2/g_1)R_1\tau_1}{1+\sigma_{21} I \frac{1}{h\nu_L}\left[\tau_2 + (g_2/g_1)\tau_1 - (g_2/g_1)A_{21}\tau_1\tau_2\right]}$$

b. This equation can be written in the standard form
$\Delta N^*(I) = \frac{\Delta N^*(0)}{1+I/I_s}$, with the saturation intensity $I_s = \frac{h\nu_L}{\sigma_{21}\tau_R}$.
Write expressions for $\Delta N^*(0)$ and the "recovery time" τ_R and give a description of what each of those terms mean physically.

3) A laser-pumped amplifier has an inversion density distribution
$\Delta N(r,z) = N_0 \exp\left[-2r^2/w_p^2 - \alpha z\right]$
a. If the total stored energy is E_{stor}, the crystal length is L, and the photon energy is $h\nu_L$, calculate an expression for N_0.
b. Let $E_0 = 50\text{mJ}$, L = 1 cm, $w_p = 2\text{mm}$, and α = 3/cm. Calculate N_0 in cm^{-3}.

c. The single-pass gain in the amplifier is given by

$G(r) = \exp\left[\sigma_{21}\int_0^L \Delta N(r,z)dz\right] = \exp\left[\Gamma_{stor}(r)/\Gamma_s\right]$, where $\Gamma_{stor}(r)$ is the stored energy fluence and $\Gamma_s = h\nu_L/\sigma_{21}$ is the saturation fluence. Calculate $\Gamma_{stor}(r)$. What parameters does the stored fluence depend on?

d. If our input seed pulse has an energy of E_0, the input energy fluence is $\Gamma(r) = \Gamma_0 \exp\left[-2r^2/w_L^2\right]$, with Γ_0 so that integrating over the beam area gives E_0. On one graph, plot the output energy fluence for w_L = 5mm and for w_L = 1mm along with the input fluence. To see how the beam is reshaped by the gain, normalize each curve so that the peak is = 1.