NL mixing with guided waves

solve linear eqn. for guided modes

$$\nabla_T^2 \vec{A}_1 + (k_0^2 n^2(r) - k_z^2)\vec{A}_n = 0$$

solutions $U_m(r)$ with k_{zm} depending on mode.

if input is in well-defined mode say U_0

source term $\sim$ $U_0(r)^q$ for q^{th} harmonic gen.

Note that transverse wave eqn. is similar to Schrödinger eqn

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r)\Psi = E\Psi$$

- $k_0^2 n^2(r)$ is an effective potential
- k_z^2 is the eigenvalue

ex. step index waveguide

is like a quantum square well

in general there are bound and unbound solutions

- concentrate on bound modes

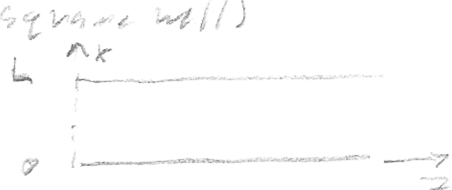
let $U_m(r)$ be an eigenfunction for waveguide

$$\nabla_T^2 U_m + k_0^2 n^2(r) U_m = k_{zm}^2 U_m$$

example: metal waveguide (like ∞ square well)

$$U_m(x) = \sin\left(\frac{m\pi}{L}x\right)$$

$$\partial_x^2 U_m = -\left(\frac{m\pi}{L}\right)^2 U_m$$



guided wave NLO

- expand E_n's (at ω_n) into spatial modes.
- coefficients a_m represent power in each mode.

→ separate equations for $a_m(z)$ for each mode, ω_n

wave eqn:

$$\frac{\partial^2}{\partial z^2} E_n + \nabla_T^2 E_n + k_{on}^2 n^2(\vec{r}) E_n = -4\pi k_{on}^2 P_n$$

- suppress vectors on fields
- $k_{on} = \omega_n / c$

expand into modes: let $\beta_{nm} = k_z$ at ω_n, mode m → "propagation constant"

$$E_n \Rightarrow \underbrace{\sum a_{nm}(z)}_{\text{power (really } \sqrt{\text{power}} \text{ at } \omega_n \text{ in mode } m)} u_{nm}(\vec{r}) e^{i(\beta_{nm} z - \omega_n t)}$$

slowly varying envelope

Consider LHS only, one mode only.

$$u_{nm}(\vec{r}) e^{i\beta_{nm} z} \left(\cancel{-\beta_{nm}^2 a_{nm}} + 2i\beta_{nm} \frac{d a_{nm}}{dz} + \frac{d^2 a_{nm}}{dz^2} + \cancel{\nabla_T^2 a_{nm} + k_0^2 n^2 a_{nm}} \right)$$

a_{nm} are eigenfunctions → cancellations

slowly-varying envelope → drop $\frac{d^2 a_{nm}}{dz^2}$

→ $2i\beta_{nm} \frac{d a_{nm}}{dz} e^{i\beta_{nm} z}$ on LHS for each mode.

example RHS: $-4\pi k_{on}^2 \chi^{(3)} E_1^3$ for 3rd harmonic

if E_1 is in one mode m'

$$\rightarrow -4\pi k_{on}^2 \chi^{(3)} a_{1m'}^3 u_{1m'}^3(\vec{r}) e^{i3\beta_{1m'} z}$$

now eigenvalue eqn reads

$$-\left(\frac{m\pi}{L}\right)^2 U_m + k_0^2 n^2(x)\, U_m = k_{zm}^2 U_m$$

which leads to an expr. for the eigenvalue:

$$k_{zm} = \sqrt{k_0^2 n^2(x) - k_x^2} \quad = \text{dispersion relation}$$

where $k_x = m\pi/L$

Note dispersion depends on mode.

An arbitrary wave can be written as a superposition

$$E_n(x,z,t) = \sum a_m U_m(x)\, e^{i(k_{zm} z - \omega t)}$$

(for metal w.g. this is Fourier series)

each mode propagates with a flat wave front at its own phase velocity $v_{ph} = \omega / k_{zm}$

Note similarity to QM:

$$\Psi(x,t) = \sum_m a_m \psi_m(x)\, e^{-iE_m t/\hbar}$$

How do we get coefficients a_m?

in QM: inner product or overlap integral

$$\int \psi_{m'}^*(x)\, \psi(x)\, dx = \sum_m a_m \int \psi_{m'}^*(x)\, \psi_m(x)\, dx$$

$$= \sum_m a_m \delta_{m'm} = a_{m'}$$

using orthogonality relation

which also assumes $\psi_m(x)$ are normalized.

Waveguides:

$$\int U_{m'}^*(x,y)\, A(x,y)\, dx\, dy = a_{m'}$$

In QM, normalization is an expression of probability:

$$\int \psi^*(x)\,\psi(x)\,dx = 1$$

in EM

$$E(x,y,z) = \sum a_m u_m(x,y)\, e^{i(k_{zm} z - \omega t)}$$

intensity (~ probability density)

$$I(x,y) = \frac{1}{2} n(x,y)\, c\, |E|^2$$

$$= \frac{1}{2} n(x,y)\, c \sum_{mm'} a_m a_{m'} u_m u_{m'} e^{i(k_{zm} - k_{zm'})z}$$

note that actual intensity profile changes w/ z

- mode beating.

Total power

$$P = \int I(x,y)\, dx\, dy$$

we construct $u_m(x,y)$ to be normalized so that

$$\int u_{m'}^*(x,y)\, u_m(x,y)\, dx\, dy = \delta_{mm'}$$

we are assuming weak guiding so that $n(x,y)$ ~ constant across beam.

otherwise define

$$\delta_{mm'} = \int u_{m'}^*(x,y)\, u_m(x,y)\, n(x,y)\, dx\, dy$$

$$P = \frac{1}{2} n_{avg} c \sum_m |a_m|^2$$

often, absorb $(\frac{1}{2} n_{avg} c)^{1/2}$ into definition of a_m so that $|a_m|^2$ represents the power in each mode.

now we have a set of coupled mode eqns

$$\sum_m \beta_{3m} \frac{da_{3m}}{dz} u_{3m}(\vec{r}) = 2\pi i k_{03}^2 \chi^{(3)} a_{1m'}^3 u_{1m'}^3(\vec{r}) e^{i3\beta_{1m'}z}$$

Finally, pick out one mode m'' $\int d^2r \, u_{3m''}(\vec{r})$ ~ on both sides.

$$\beta_{3m''} \frac{da_{3m''}}{dz} = 2\pi i k_{03}^2 \chi^{(3)} a_{1m'}^3 \underbrace{\left[\int u_{3m''} u_{1m'}^3 \, d^2r\right]}_{\text{overlap integral}} e^{i\Delta\beta z}$$

$$\Delta\beta = 3\beta_{1m'} - \beta_{3m''}$$

overlap integral becomes a weighting factor on $\chi^{(3)}$

phasematching is <u>mode-specific</u>.

Approach:
- try to phase match mode combination w/ best overlap

Example: THG in capillary waveguides. see slides.