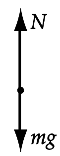
**2-25.**

**a)** At A, the forces on the ball are:




The track counters the gravitational force and provides centripetal acceleration

$$N - mg = mv^2/R$$

Get $v$ by conservation of energy:

$$E_{top} = T_{top} + U_{top} = 0 + mgh$$

$$E_A = T_A + U_A = \frac{1}{2}mv^2 + 0$$

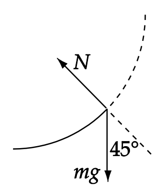
$$E_{top} = E_A \rightarrow v = \sqrt{2gh}$$

So

$$N = mg + m2gh/R$$

$$\boxed{N = mg\left(1 + \frac{2h}{R}\right)}$$

**b)** At $B$ the forces are:

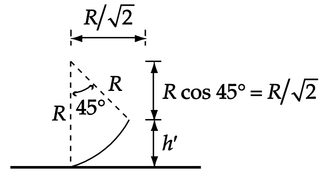



$$N = mv^2/R + mg\cos 45°$$

$$= mv^2/R + mg/\sqrt{2} \qquad (1)$$

Get $v$ by conservation of energy. From a), $E_{total} = mgh$.

At $B$, $E = \frac{1}{2}mv^2 + mgh'$




$$R = \frac{R}{\sqrt{2}} + h' \quad \text{or} \quad h' = R\left(1 - \frac{1}{\sqrt{2}}\right)$$

So $E_{total} = T_B + U_B$ becomes:

$$mgh = mgR\left(1 - \frac{1}{\sqrt{2}}\right) + \frac{1}{2}mv^2$$

Solving for $v^2$

$$2\left[gh - gR\left(1 - \frac{1}{\sqrt{2}}\right)\right] = v^2$$

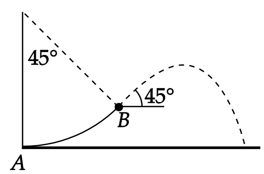
Substituting into (1):

$$N = mg\left[\frac{2h}{R} + \left(\frac{3}{\sqrt{2}} - 2\right)\right]$$

**c)** From b) $v_B^2 = 2g\left[h - R + R/\sqrt{2}\right]$

$$\boxed{v = \left[2g\left(h - R + R/\sqrt{2}\right)\right]^{1/2}}$$

**d)** This is a projectile motion problem




Put the origin at $A$.

The equations:

$$x = x_0 + v_{x0}t$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

become

$$x = \frac{R}{\sqrt{2}} + \frac{v_B}{\sqrt{2}}t \qquad (2)$$

$$y = h' + \frac{v_B}{\sqrt{2}}t - \frac{1}{2}gt^2 \qquad (3)$$

Solve (3) for $t$ when $y = 0$ (ball lands).

$$gt^2 - \sqrt{2}\,v_B t - 2h' = 0$$

$$t = \frac{\sqrt{2}\,v_B \pm \sqrt{2v_B^2 + 8gh'}}{2g}$$

We discard the negative root since it gives a negative time. Substituting into (2):

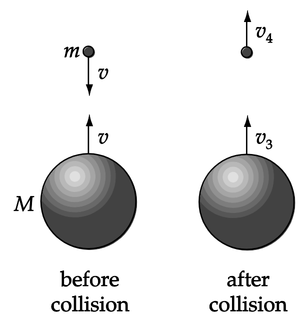
$$x = \frac{R}{\sqrt{2}} + \frac{v_B}{\sqrt{2}}\left[\frac{\sqrt{2}\, v_B \pm \sqrt{2v_B^2 + 8gh'}}{2g}\right]$$

Using the previous expressions for $v_B$ and $h'$ yields

$$\boxed{x = \left(\sqrt{2} - 1\right) R + h + \left[h^2 - \frac{3}{2} R^2 + \sqrt{2}\, R^2\right]^{1/2}}$$

**e)** $U(x) = mgy(x)$, with $y(0) = h$, so $U(x)$ has the shape of the track.

**2-28.**




The problem, as stated, is completely one-dimensional. We may therefore use the elementary result obtained from the use of our conservation theorems: energy (since the collision is elastic) and momentum. We can factor the momentum conservation equation

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4 \tag{1}$$

out of the energy conservation equation

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_3^2 + \frac{1}{2} m_2 v_4^2 \tag{2}$$

and get

$$v_1 + v_3 = v_2 + v_4 \tag{3}$$

This is the "conservation" of relative velocities that motivates the definition of the coefficient of restitution. In this problem, we initially have the superball of mass *M* coming up from the ground with velocity $v = \sqrt{2gh}$ , while the marble of mass *m* is falling at the same velocity. Conservation of momentum gives

$$Mv + m(-v) = Mv_3 + mv_4 \tag{4}$$

and our result for elastic collisions in one dimension gives

$$v + v_3 = (-v) + v_4 \tag{5}$$

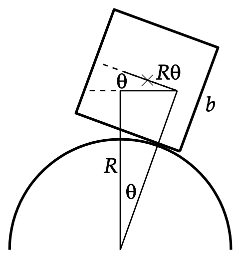
solving for $v_3$ and $v_4$ and setting them equal to $\sqrt{2gh_{item}}$ , we obtain

$$h_{marble} = \left[\frac{3-\alpha}{1+\alpha}\right]^2 h \qquad (6)$$

$$h_{superball} = \left[\frac{1-3\alpha}{1+\alpha}\right]^2 h \qquad (7)$$

where $\alpha \equiv m/M$. Note that if $\alpha < 1/3$, the superball will bounce on the floor a second time after the collision.

**2-42.**




From the figure, we have $h(\theta) = (R + b/2)\cos\theta + R\theta\sin\theta$, and the potential is $U(\theta) = mgh(\theta)$. Now compute:

$$\frac{dU}{d\theta} = mg\left[-\frac{b}{2}\sin\theta + R\theta\cos\theta\right] \qquad (1)$$

$$\frac{d^2U}{d\theta^2} = mg\left[\left(R - \frac{b}{2}\right)\cos\theta - R\theta\sin\theta\right] \qquad (2)$$

The equilibrium point (where $dU/d\theta = 0$) that we wish to look at is clearly $\theta = 0$. At that point, we have $d^2U/d\theta^2 = mg\left(R - b/2\right)$, which is stable for $R > b/2$ and unstable for $R < b/2$. We can use the results of Problem 2-46 to obtain stability for the case $R = b/2$, where we will find that the first non-trivial result is in fourth order and is negative. We therefore have an equilibrium at $\theta = 0$ which is stable for $R > b/2$ and unstable for $R \le b/2$.