Assignment 6 solutions
PHGN361

Homework solutions

1. This applet illustrates separation of variables used in the Schrodinger equation for a harmonic oscillator. By moving the slider you can add different solutions to match some initial condition (gaussian wavepacket). This is the same process in adding separation of variables solutions to Laplace's equation to match boundary conditions. Note that probability density is plotted in this applet, not the solutions to the Schrodinger equation directly.

2. Gauss's law is used at the surface of the 100 V conductor to determine the charge density on the surface. Choose a small Gaussian pillbox with sides perpendicular to the surface and endcaps parallel to the surface, one endcap of which is embedded in the conductor (E=0) while the other is just outside the conductor. This encloses the surface charge σdA. Gauss's law then becomes $EdA = \sigma dA/\epsilon_0$. Solving for $E = \sigma/\epsilon_0$. However, the relaxation method calculates not E but rather $E = -\vec{\nabla}V = -(\hat{x}\partial^2 V/\partial x^2 + \hat{y}\partial V/\partial y + \hat{z}\partial V/\partial z)$. E can then be obtained numerically by finding $\Delta V/\Delta x$ just above the surface where x is in the direction normal to the surface. That is find V in a cell just above the surface and obtain $\Delta V = V_{above} - 100V$ with $\Delta x =$ the distance between cells. Do this all along the surface to map out the charge density.

3. Choose a fixed voltage for each conductor. Apply the relaxation method to find the voltage. Find the electric field everywhere using the gradient numerically (see above). For each cell find $\epsilon_0 E^2/2$ (the energy density in $Joules/m^3$) and multiply this by the volume associated with each cell. Do this for all cells and sum over all cells to find the energy of this configuration (energy required to assemble it and therefore energy that can be extracted from it). Now change the voltage on conductor 1 while that of the other remains fixed and repeat the calculation. Do this for a range of voltages on conductor 1. Then change the voltage on conductor 2 and repeat the variation on conductor 1. Construct a table of configuration energies for the differing voltages. The minimum in this table yields the voltages on each conductor. This is truly a job for a computer.

4. Again sum the energy density over the volume.