MATH348 - March 1, 2010
Exam I - 50 Points - 50 minutes

NAME:
SECTION:

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points)
(a) True/False : Mark each statement as either true or false.
i. Suppose that $\mathbf{A x}=\mathbf{b}$, where $\mathbf{A} \in \mathbb{R}^{n \times n}$, has no solutions. The corresponding homogeneous system, $\mathbf{A x}=\mathbf{0}$, has only the trivial, $\mathbf{x}=\mathbf{0}$, solution.
ii. If $\mathbf{A} \in \mathbb{R}^{m \times n}$ has a row of zeros then $\mathbf{A x}=\mathbf{0}$ always has infinitely-many solutions.
iii. It is impossible for a vector to be in both the null-space and column-space of a matrix.
iv. If the dimension of the column-space of $\mathbf{A}_{n \times n}$ is $n$ then $\mathbf{A x}=\mathbf{0}$ has only the trivial solution.
v. The system $\mathbf{A x}=\mathbf{0}$, where $\mathbf{A} \in \mathbb{R}^{3 \times 4}$ has only the trivial solution.
(b) Short Response : Provide a short justification of your conclusion.
i. Suppose $\mathbf{V}_{n \times n}$ is a matrix whose columns form a basis for $\mathbb{R}^{n}$. What can be said about the determinant of $\mathbf{V}$ ?
ii. Suppose that $\lambda=0$ is an eigenvalue of $\mathbf{A}$. What can be said about $\mathbf{A}^{-1}$ ?
2. (10 Points) Quickies
(a) Given,

$$
\left[\begin{array}{ll|l}
1 & 3 & 2 \\
3 & h & k
\end{array}\right]
$$

Determine all values of $h$ and $k$ so that the system has:
i. Exactly one solution
ii. Infinitely-many solutions
iii. No solutions
(b) Find all eigenvalues of,

$$
\mathbf{A}=\left[\begin{array}{rrr}
4 & 0 & 1 \\
-2 & 1 & 0 \\
-2 & 0 & 1
\end{array}\right]
$$

(c) Find all values of $h$ so that the following vectors are linearly independent.

$$
\mathbf{x}=\left[\begin{array}{r}
1 \\
-1 \\
-3
\end{array}\right], \quad \mathbf{y}=\left[\begin{array}{r}
-5 \\
7 \\
8
\end{array}\right], \quad \mathbf{z}=\left[\begin{array}{l}
1 \\
1 \\
h
\end{array}\right]
$$

(d) Find the general solution to the following linear system of equations.

$$
\begin{array}{r}
x_{2}+2 x_{3}=0 \\
4 x_{1}+5 x_{2}+6 x_{3}=0 \\
8 x_{1}+9 x_{2}+10 x_{3}=0
\end{array}
$$

3. (10 Points) Find a basis for the null-space and column-space of,

$$
\mathbf{A}=\left[\begin{array}{rrrrr}
2 & -3 & 6 & 2 & 5 \\
0 & 0 & 3 & -1 & 1 \\
0 & 0 & 0 & 1 & 3 \\
0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

4. (10 Points) Let $\mathbf{A}=\left[\begin{array}{ll}3 / 4 & 1 / 4 \\ 1 / 4 & 3 / 4\end{array}\right]$.
(a) Find the eigenvalues of $\mathbf{A}$.
(b) Find the eigenvectors of $\mathbf{A}$.
(c) Calculate $\lim _{n \rightarrow \infty} \mathbf{A}^{n}$.
5. (10 Points) Suppose that A has the following eigenvalue, eigenvector pairs.

$$
\begin{aligned}
& \lambda_{1}=0, \quad \mathbf{e}_{1}=\left[\begin{array}{l}
1 \\
0 \\
1
\end{array}\right] \\
& \lambda_{2}=1, \quad \mathbf{e}_{2}=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right] \\
& \lambda_{3}=-1, \quad \mathbf{e}_{3}=\left[\begin{array}{r}
-1 \\
0 \\
1
\end{array}\right]
\end{aligned}
$$

Find the general solution to $\mathbf{A x}=\mathbf{0}$.

