Note Title 9/11/2006

Forced, damped simple harmonic osc.

$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = F/m \qquad F(t)$

$\gamma \equiv$ damping coefficient $[\gamma] = \frac{1}{\text{time}}$

$\omega_0^2 = \frac{k}{m}$

simplify by letting $F(t) \rightarrow f_0 e^{i\omega t}$

See Later: if we can solve for $F = e^{i\omega t}$ then we can solve for any $F(t)$ Fourier Series

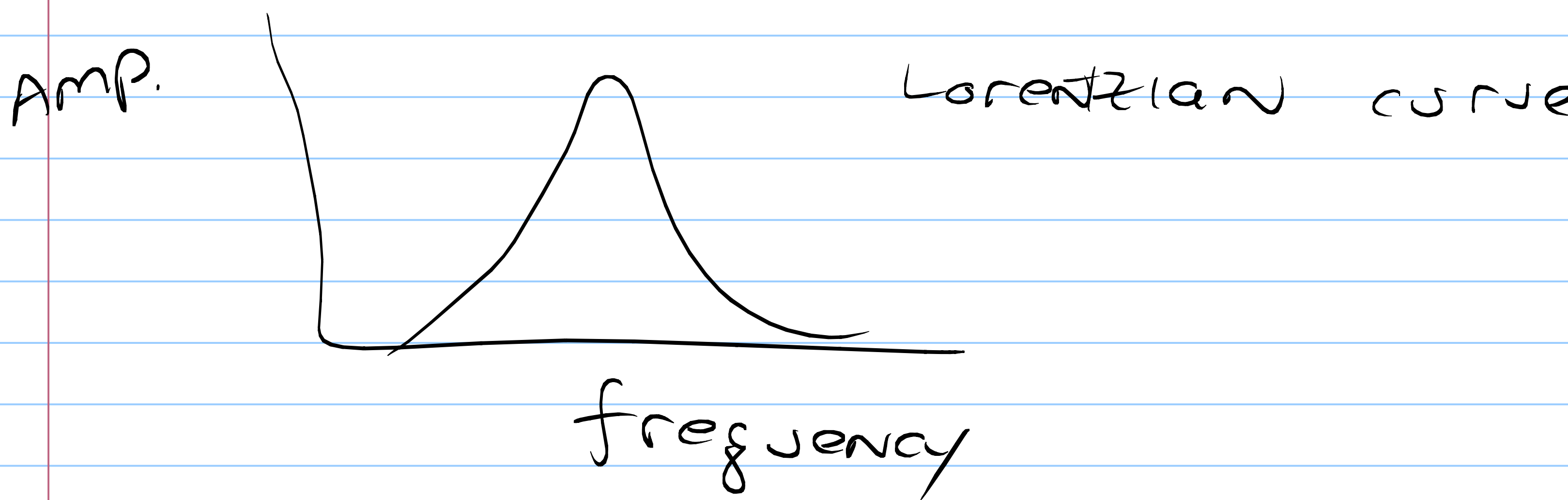
$$x(t) = x_0 e^{i\omega t}$$

$$x_0 = \frac{F_0/m}{\underbrace{\left[(\omega_0^2 - \omega^2) + I\gamma\omega\right]}}$$

$\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$ mag. of denom.

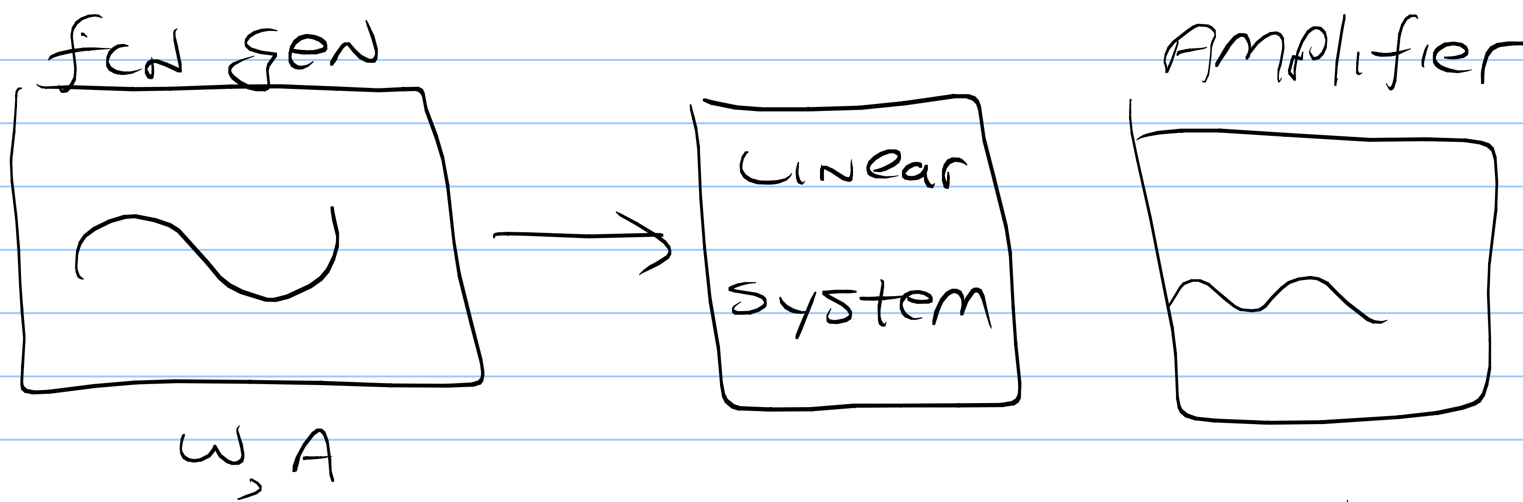
phase of denom: $\tan^{-1}\left[\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right]$

$$x(t) = e^{i\omega t}\left[\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}}\, e^{-i\tan^{-1}\left[\frac{\gamma\omega}{\omega_0^2-\omega^2}\right]}\right]$$



Amplitude Spectrum

Phase spectrum



spectroscopy.

$$e^{ix} = \cos x + i\sin x$$

Euler

$$(e^{ix})^2 = e^{i2x} = \cos 2x + i\sin 2x$$

$$(e^{ix})^n = \cos nx + i\sin nx$$

$$e^{-ix} = \cos x - i\sin x$$

$$e^{ix} + e^{-ix} = 2\cos x$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$e^{ix} - e^{-ix} = 2i\sin x$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$ inverse Euler

$$e^{ix} = \cos x + i\sin x$$

suppose $x = iy$ y real

$$e^{-y} = \cos(iy) + i\sin(iy)$$

$x = iy \qquad y$ real

$$\frac{e^{ix} + e^{-ix}}{2} = \cos x \qquad \frac{e^{ix} - e^{-ix}}{2i} = \sin x$$

$x \to iy$

$$\frac{e^{-y} + e^{y}}{2} = \cos(iy) \qquad \frac{e^{-y} - e^{y}}{2i} = \sin iy$$

$$\frac{e^{y} + e^{-y}}{2} = \cos(iy) \qquad i\,\frac{e^{y} - e^{-y}}{2} = \sin iy$$

$$\cos(iy) = \cosh(y) \qquad -i\sin(iy) = \sinh(y)$$