1.1.6 Following is a statement of a theorem which can be proven using calculus or precalculus mathematics. For this theorem, a, b, and c are real numbers.

Theorem If f is a quadratic function of the form $f(x) = ax^2 + bx + c$ and $a < 0$, then the function f has a maximum value when $x = \dfrac{-b}{2a}$.

Using only this theorem, what can be concluded about the functions given by the following formulas?

1. $g(x) = -8x^2 + 5x - 2$
2. $f(x) = -4x^2 - 3x + 7$
3. $F(x) = -x^4 + x^3 + 9$

1.1.7 Following is a statement of a theorem which can be proven using the quadratic formula. For this theorem, a, b, and c are real numbers.

Theorem If f is a quadratic function of the form $f(x) = ax^2 + bx + c$ and $ac < 0$, then the function f has two x-intercepts.

Using only this theorem, what can be concluded about the functions given by the following formulas?

1. $g(x) = -8x^2 + 5x - 2$
2. $f(x) = -4x^2 - 3x + 7$
3. $F(x) = -x^4 + x^3 + 9$

1.1.8 Following is a statement of a theorem about certain cubic equations. For this theorem, b represents a real number.

Theorem A If f is a cubic function of the form $f(x) = x^3 - x + b$ and $b > 1$, then the function f has exactly one x-intercept.

Following is another theorem about x-intercepts of functions:

Theorem B If f and g are functions with $g(x) = k \cdot f(x)$ where k is a nonzero real number, then f and g have exactly the same x-intercepts.

Using only these two theorems and some simple algebraic manipulations, what can be concluded about the functions given by the following formulas?

1. $f(x) = x^3 - x + 7$
2. $r(x) = x^4 - x + 11$
3. $F(x) = 2x^3 - 2x + 7$

1.2.4 Construct a know-show table and write a complete proof for the following statement:

a. If m is an even integer, then $5m + 7$ is an odd integer.

1.2.5 Construct a know-show table and write a complete proof for the following statement:

b. If m is an odd integer, then $3m^2 + 7m + 12$ is an even integer.

2.2.6 Prove the logical equivalency, $[P \vee Q) \rightarrow R] \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$.

2.2.12 Let x be a real number. Consider the following conditional statement:

$$\text{If } x^3 - x = 2x^2 + 6, \text{ then } x = -2 \text{ or } x = 3$$

Which of the following statements have the same meaning as this conditional statement and which ones are negations of this conditions statement? Explain each conclusion.

(a) If $x \neq -2$ and $x \neq 3$, then $x^3 - x \neq 2x^2 + 6$

(b) If $x = -2$ or $x = 3$, then $x^3 - x = 2x^2 + 6$

(c) If $x \neq -2$ or $x \neq 3$, then $x^3 - x \neq 2x^2 + 6$

(d) If $x^3 - x = 2x^2 + 6$ and $x \neq -2$, then $x = 3$

(e) If $x^3 - x = 2x^2 + 6$ or $x \neq -2$, then $x = 3$

(f) $x^3 - x = 2x^2 + 6$, and $x \neq -2$, and $x \neq 3$

(g) $x^3 - x \neq 2x^2 + 6$ or $x = -2$ or $x = 3$