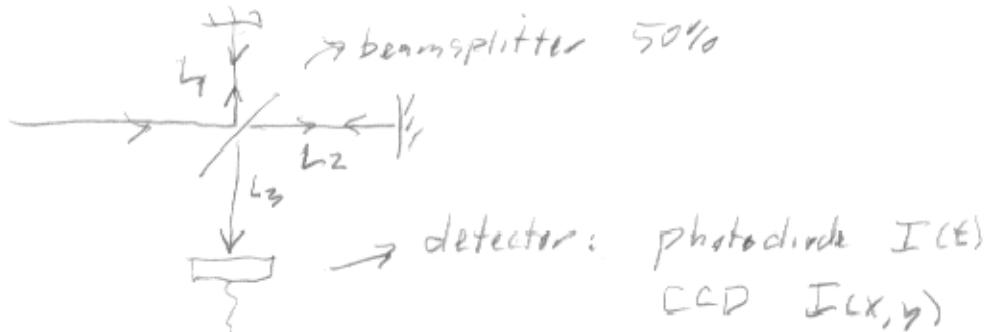


## Michelson interferometer



output field is sum of fields from paths 1 and 2:

$$\vec{E}_{\text{out}} = \vec{E}_{\text{in}} \left( e^{ik(2L_1 + L_3)} + e^{-ik(2L_2 + L_3)} \right)$$

$$= \vec{E}_1 + \vec{E}_2 \quad \text{note both have } e^{-i\omega t}$$

intensity or power at detector  $\propto \langle |\vec{E}_{\text{out}}|^2 \rangle$

$$\begin{aligned} \langle |\vec{E}_{\text{out}}|^2 \rangle &= \vec{E}_{\text{out}} \cdot \vec{E}_{\text{out}}^* \\ &= (\vec{E}_1 + \vec{E}_2) \cdot (\vec{E}_1^* + \vec{E}_2^*) \\ &= |E_1|^2 + |E_2|^2 + \vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_1^* \cdot \vec{E}_2 \end{aligned}$$

convenient trick: for complex numbers

$$z + z^* = a + ib + a - ib = 2a = 2 \operatorname{Re}(z)$$

$$z - z^* = 2 \operatorname{Im}(z)$$

$$\langle |\vec{E}_{\text{out}}|^2 \rangle = |E_1|^2 + |E_2|^2 + \underbrace{2 \operatorname{Re}(\vec{E}_1 \cdot \vec{E}_2^*)}_{\text{interference term.}}$$

- only like polarization states interfere.

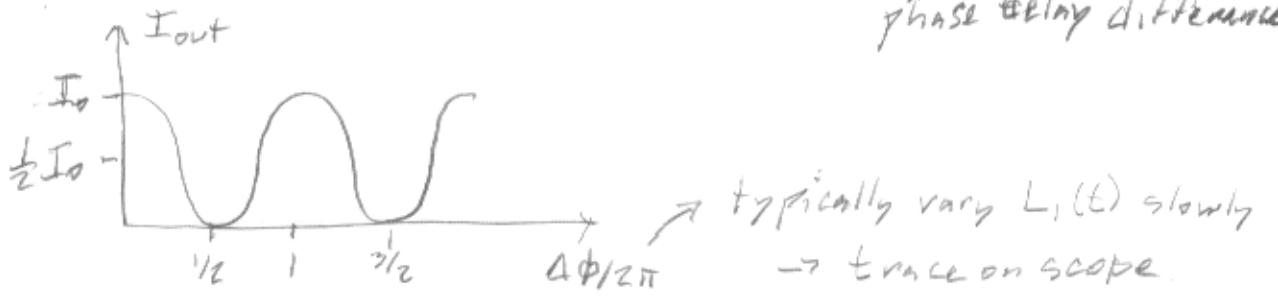
- in this case:  $I_1 = I_2$ , parallel polarization

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)I_0 = \frac{1}{4}I_0 \quad E_1 = \sqrt{I_0/4} e^{i\phi_1}$$

$$\rightarrow I_{\text{out}} = \frac{1}{4}I_0 + \frac{1}{4}I_0 + 2 \cdot \sqrt{\frac{I_0}{4}} / \sqrt{\frac{I_0}{4}} \operatorname{Re}(e^{i(\phi_1 - \phi_2)})$$

$$= \frac{1}{2}I_0 (1 + \underline{\cos(\phi_1 - \phi_2)})$$

$$\begin{aligned}
 \text{here } \phi_1 - \phi_2 &= k(2L_1 + L_3) - k(2L_2 + L_3) \\
 &= 2k(L_1 - L_2) = k_o \cdot (\text{optical path diff}) \\
 \text{common path cancels} &= w \Delta \tau \\
 &\qquad\qquad\qquad \text{phase delay difference}
 \end{aligned}$$



one full wave:  $\Delta\phi = 2\pi$  bright fringes =  $\lambda$  separat.

Where does power go when  $I_{\text{out}} = 0$ ?

→ back to input



but they both have same path!

Mach-Zehnder:



$$I_A = ?$$

$$E_{1A} = E_0 \cdot t \cdot r \cdot e^{-ikL_1}$$

$$E_{2A} = E_0 \cdot r \cdot t \cdot e^{-ikL_2}$$

$$2 \operatorname{Re}(E_{1A} E_{2A}^*) = 2E_0^2 t r \cos(k(L_1 - L_2))$$

$$\text{For 50% BS } t = \frac{1}{\sqrt{2}}, r = -\frac{1}{\sqrt{2}} \rightarrow -E_0^2 \cos \Delta\phi$$

$$E_{1B} = E_0 t \cdot t \cdot e^{ikL_1}$$

$$E_{2B} = E_0 r \cdot r \cdot e^{ikL_2}$$

$$2 \operatorname{Re}(E_{1B} E_{2B}^*) \rightarrow +E_0^2 \cos \Delta\phi$$

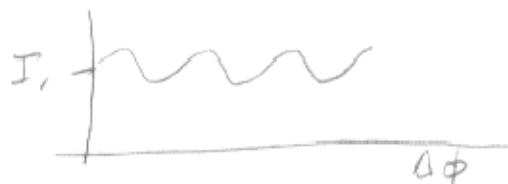
points A, B are complements of each other.

What if  $|E_1| \neq |E_2|$ ?

$$I_{\text{out}} = I_1 + I_2 + 2\sqrt{I_1} \sqrt{I_2} \cos A\phi$$

Suppose  $I_2 = \frac{1}{100} I_1$

$$\begin{aligned} I_{\text{out}} &= I_1 \left( 1 + \frac{1}{100} + 2 \cdot 1 \cdot \frac{1}{10} \cos A\phi \right) \\ &= I_1 (1.01 + 0.2 \cos A\phi) \end{aligned}$$



less contrast in fringes  
than when balanced.

Fringe visibility:

$$\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{1.21 - 0.81}{2.02} \approx 0.2$$

Notice that a mix of 1% of the energy  $\rightarrow$  20% fringes!

Scan  $L_1(t)$  with different source w?

$$I_{\text{out}}(L_1) = \frac{1}{2} I_0 \left( 1 + \cos \left( \frac{2\pi}{\lambda} (L_1 - L_2) \right) \right)$$

Ident: At  $L_1 = L_2$ :  $I_{\text{out}} = I_0$  (constructive interference)  
independent of  $\lambda$

Period of fringes depends on  $\lambda$



Expt: must use a compensated path to make sure each beam passes through same material, same mirrors.  
e.g. beam splitters are often coated on one side



## Spatial interferometry

consider same expt, with CCD detector or a screen



with our ideal plane waves, whole screen is light or dark as path is varied.

tilt the beams (adjust mirrors)

$$\text{at output: } E_1 = \frac{1}{2} E_0 e^{-i(k_x x + k_z z)} \quad \text{where } z \rightarrow 2L_1 + L_2$$

$$k_x = k_0 \sin \theta_1$$

what is different is an extra  $k_z = k_0 \cos \theta_1$

$$\phi_1(x) = k_0 \sin \theta_1, x = \text{spatial phase.}$$

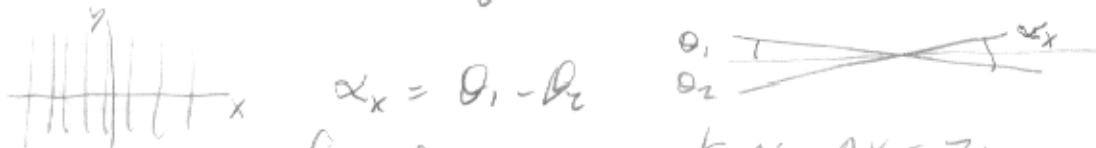
if we align the paths to be equal ( $L_1 = L_2$ ), output is

$$I_{\text{out}}(x) = \frac{1}{2} I_0 (1 + \cos [k_0 (\sin \theta_1 - \sin \theta_2)x])$$

\* time delay mapped to space: no need to scan.

to get fringes w/  $\Delta x \approx 1 \text{ mm}$ ,  $\Delta \theta$  must be small

$$\rightarrow I_{\text{out}}(x) = \frac{1}{2} I_0 (1 + \cos k_0 \alpha_x x)$$



$$\text{for } \Delta x = 1 \text{ mm: } k_0 \alpha_x \cdot \Delta x = 2\pi$$

$$\alpha_x = \lambda / \Delta x \approx 10^{-3} \text{ rad.}$$

Tilt in x and y:

$$E_1 = \frac{1}{2} E_0 e^{-i k_0 \alpha_x x} e^{i k_0 \alpha_y y}$$

for  $\lambda = 1 \text{ nm}$

$$I_{\text{out}} = \frac{1}{2} I_0 (1 + \cos k_0 (\alpha_x x + \alpha_y y))$$

→ rotated fringes

