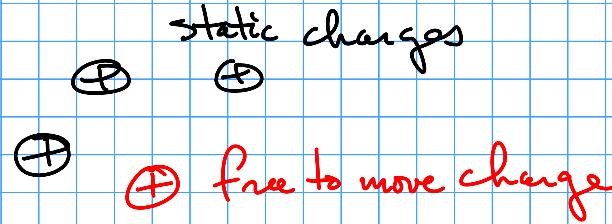
Electric Field Hockey:



Fundamental Principle:

$$\sum_i \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

due to free charge

$$q\sum_i \vec{E}_i = m\frac{d\vec{v}}{dt} \qquad \text{ODE} \qquad \vec{E}_{tot}(x,y) = \sum_i \vec{E}_i$$

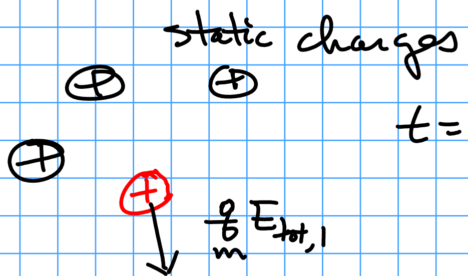
due to static charges

$$\vec{v}(x,y,t)$$

plus initial conditions $\vec{v}_0(x_0, y_0, t_0)$ yields unique soln.

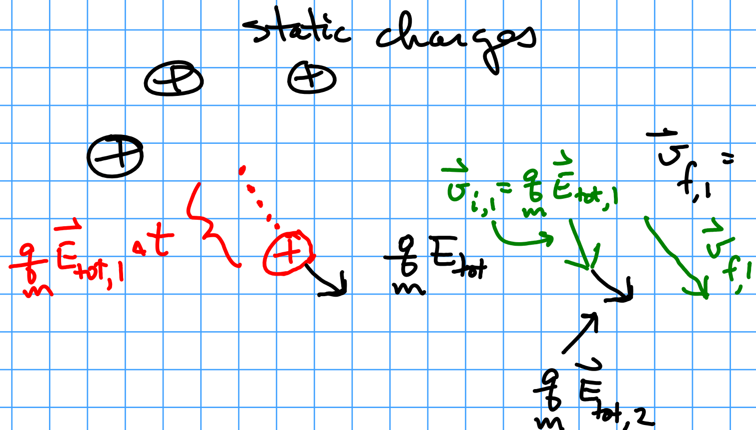
Method

$$\frac{q}{m}\vec{E}_{tot}(x,y) = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \qquad \vec{v}_f = \frac{q}{m}\vec{E}_{tot}(x,y)\Delta t + \vec{v}_i$$

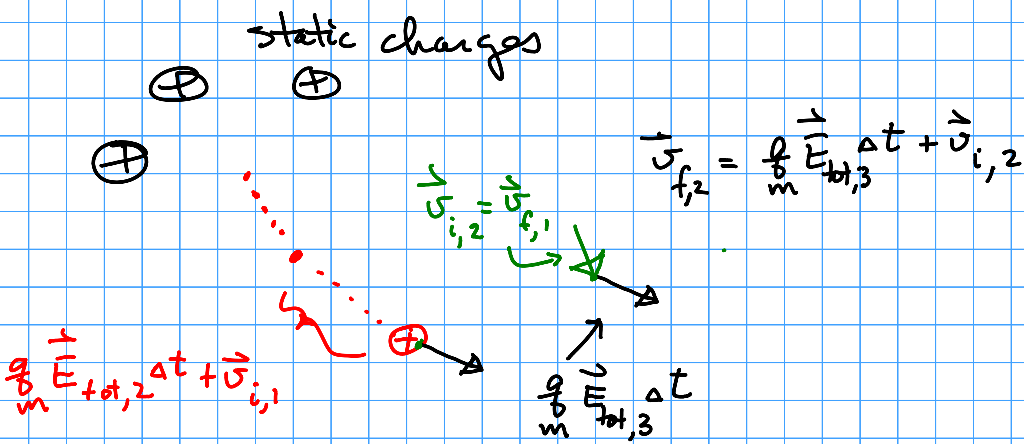


$t = 0 \quad \vec{v}_i = 0$

$t = \Delta t$



$$\vec{v}_{f,1} = \frac{q}{m}\vec{E}_{tot,2}\Delta t + \vec{v}_{i,1}$$



- Does charge always move along field line?

- Can a particle be trapped?

$\boxed{\text{Overview of Chapter 5}}$

- Lorentz Force

$$\vec{F} = q\vec{v}\times\vec{B} \longrightarrow \int I\, d\vec{\ell}\times\vec{B} \longrightarrow \;?\; \longrightarrow \;?$$

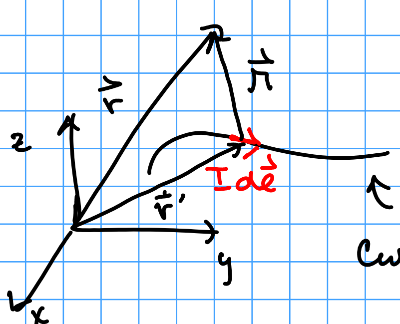
pt charge ⟶ wire ⟶ sheet of moving charges ⟶ volume of moving charges

Need to find $\vec{B}$: assume magnetostatics ≡ steady currents

Biot & Savart

$$\vec{B} = \frac{\mu_0}{4\pi}\int I\,\frac{d\vec{\ell}\times\hat{\mathscr{r}}}{\mathscr{r}^2} \longrightarrow \;?\; \longrightarrow \;?$$

⟶ sheet of moving charges ⟶ volume of moving charges



current carrying wire

Electrostatics

$$\begin{tikzcd} & dQ \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\ dV \arrow[rr, leftrightarrow] & & d\vec{E} \end{tikzcd}$$

magnetostatics

$$\begin{tikzcd} & dI \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\ d\vec{A} \arrow[rr, leftrightarrow] & & dB \end{tikzcd}$$

$\lambda \quad \frac{C}{m}$

$+ + + + + + + \longrightarrow v$ Scalar

$$I = \frac{dq}{dt} \qquad dq = \lambda\, dl \qquad \frac{dq}{dt} = \lambda \underbrace{\frac{d\vec{l}}{dt}}_{\vec{v}}$$

$$\vec{I} = \lambda \vec{v}$$

λ

$+ + + + + + + + \longrightarrow v$

$v \longleftarrow - - - - - - - -$

$-\lambda$

$$\vec{I} = +\lambda v \hat{x} + -\lambda v(-\hat{x}) = 2\lambda v \hat{x}$$

$$d\vec{F} = dq\, \vec{v} \times \vec{B} = \lambda\, dl\, \vec{v} \times \vec{B} = I\, d\vec{l} \times \vec{B}$$

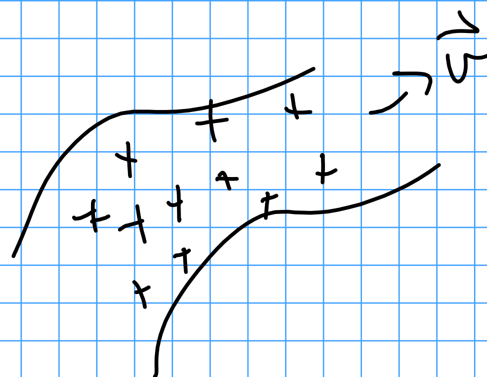
$$I = \frac{dq}{dt} = \lambda \frac{dl}{dt}$$

$$d\vec{F} = \vec{I} \times \vec{B}\, dl$$

no vector

$$\vec{F} = \int I(d\vec{l} \times \vec{B}) = \int (\vec{I} \times \vec{B})\, dl$$

Surface current



$$dq = \sigma\, da$$

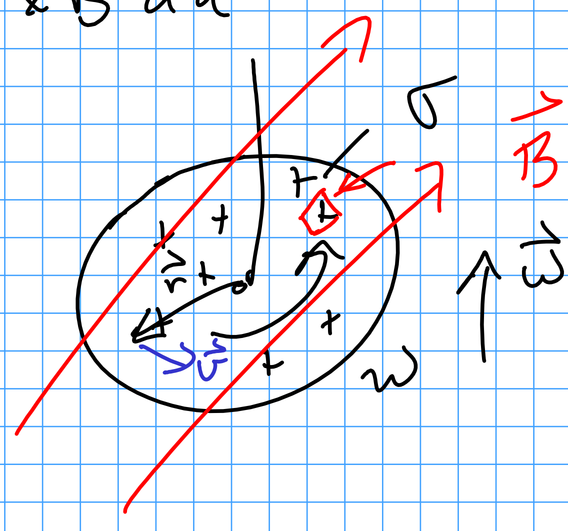
$$d\vec{F} = dq\, \vec{v} \times \vec{B} = \sigma\, da\, \vec{v} \times \vec{B} = \vec{K} \times \vec{B}\, da$$

$$\vec{K} = \sigma \vec{v}$$

$$da = r\, d\phi\, dr$$

$$\vec{F} = \int \vec{K} \times \vec{B}\, da$$

second



$$\vec{K} = \sigma \vec{v}$$

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega r \sin 90\, \hat{\phi} = \omega r \_\, \hat{x} + \_\, \hat{y}$$

$$\vec{K} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ & & \\ \omega r\_ & \omega r\_ & 0 \end{vmatrix}$$

$$\begin{vmatrix} B_x & B_y & B_z \end{vmatrix}$$

Why use cartesian coords to calculate $\vec{F} = \int \vec{K} \times \vec{B} \, da$?

ANSWER: ONLY $\hat{x}, \hat{y}, \hat{z}$ are constant & can come outside the integral!

$da = r \, d\varphi \, dr$ & $\hat{\varphi}, \hat{r}$ vary with r & φ