E. Kreyszig, Advanced Engineering Mathematics, 8^{th} **ed.**	Section N/A, pgs. N/A
Lecture: Overview and Outline	Module: 01
Suggested Problem Set: {NULL}	January 7, 2009

Quote of Lecture 1	
Montgomery Burns: I'll keep it short and sweet. Family, religion, friendship. These are the three demons you must slay if you wish to succeed in business.	
	The Simpsons: The Old Man and the Lisa (1997)

The course Advanced Engineering Mathematics serves the following CSM disciplines,

- Engineering (Civil, Electrical, Environmental, Mechanical)
- Geophysics
- Rouge Physicists
- Students pursuing an ASI or minor in Mathematics

by introducing them to concepts from,

- Linear Algebra
- Partial Differential Equations

in order to connect their two-years of post secondary mathematics to the rich field of applied mathematical modeling.

If mathematics is the study of the meaning and properties associated with the symbolic formalism then applied mathematical modeling is the application of this knowledge to real-world phenomenon in an effort to draw non-experimental conclusions. The ultimate goal is to make predictions about natural occurrences in order to gain control over them.[1] Since this has been going on for most of human existence the body of material is massive and deep. We will be mostly concerned with calculation, but if we remember to also concern ourselves with the mathematical roots we will achieve a more comprehensive and thus connected understanding enabling us to remember more concepts for a longer period of time.

It is my perspective that the key point of this material is to draw as many conclusions about the symbols, which naturally arise as solutions to certain differential equations, as possible. This is by no means a small task and there are many different and equally justifiable routes to this goal. However, this goal, I feel, is best served by studying first the straight-forward concepts of linear algebra and connect these concepts to the more complicated study of linear partial differential equations used to model ideal, flows, vibrations, and potential fields. [2] [3]

My opinion is largely due to the fact that linear algebra can abstract a method, which you should already be aware of, to any finite number of directions. That is, for certain two-dimensional linear problems say,

$$\frac{d\mathbf{Y}}{dt} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\mathbf{Y}, \quad \mathbf{Y}(0) = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, \tag{1}$$

[1] For example, it is sometimes difficult to construct experiments involving complicated fluid flow but it is 'easier' to write down mathematical models for these flows and evaluate them under 'experimental conditions'. A reason for this might be to understand how to mix two fluids into one fluid while using the least amount of energy.

[2] I must note that it is highly useful to study the theory of linear algebra as it is the most applicable mathematical tool in the sciences and well worth a stand alone course. Those interested should consider taking MATH332-Linear Algebra.

[3] The concept of linearity is a powerful tool that allows one to say that an object with a certain property is equivalent to the sum of of other objects having the same property. This is not generally true of nonlinear phenomenon.

one can construct solutions via *linear combinations* in some *eigenbasis* of the constant coefficient matrix. Specifically, solutions takes the form,

$$\mathbf{Y}(t) = k_1\mathbf{v}_1 e^{\lambda_1 t} + k_2\mathbf{v}_2 e^{\lambda_2 t}, \tag{2}$$

for appropriate choices of $k_1, k_2, \lambda_1, \lambda_2, \mathbf{v}_1, \mathbf{v}_2$. [4] This concept is deep and difficult to pick out when one is constructing solutions in infinite-dimensional spaces. [5] So, we start with the a study of linear algebra, building on your previous work with differential equations and vector calculus, so that when we finish with a survey of PDE's the mathematics will have a better 'sense'.

For example, we will find out later on that for solutions to the linear PDE,

$$\frac{\partial u}{\partial t} = c^2 \triangle u, \tag{3}$$

whose unknown function u has spatial component defined on a closed and bounded domain in $\mathbb{R}$, can take the following form,

$$u(x,t) = \sum_{n=-\infty}^{\infty} k_n e^{-i\omega_n x} e^{-(c\omega_n)^2 t}, \tag{4}$$

for appropriate choice of $k_i, w_i,\ i \in \mathbb{N}$. It is not obvious what this summation means and how it could possibly be related to the study of the PDE, which models the flow of some density. It may or may not be obvious that this is a linear combination of *basis vectors*, or that since we used infinitely many *basis vectors* the summability of the series should be in question.[6] However, if we start small then we will have the experience needed to be comfortable with statements like (4) and be able to concentrate on understanding what they can tell us about solutions to (3). I have delivered this material starting at linear algebra building to Fourier series and PDE's and, though this ends with the courses most complicated calculations, students performed better than in previous semesters.

MATH348 - Advanced Engineering Mathematics - Course Goals

- Understand the vocabulary, methods and applications of linear algebra, Fourier analysis and partial differential equations.
- Study the algebraic structure of linear spaces, which gives a systematic method for creating solutions to linear problems, in order to strengthen concepts and interconnections of the course material.

MATH348 - Advanced Engineering Mathematics - Course Objectives

- Practice the row elimination algorithm applied to $\mathbf{Ax} = \mathbf{b}$ to find solutions and interpret them geometrically in terms of linear combinations of arbitrary basis vectors of the underlying linear vector space.
- Learn to calculate the spectrum of a square-matrix associated with the transformation equation $\mathbf{Ax} = \lambda\mathbf{x}$ and from this construct spectral decompositions of symmetric matrices.
- Use orthogonality to construct Fourier representations of 'arbitrary' functions on $\mathbb{R}$ and relate these linear combinations to signal analysis.
- Solve physically relevant linear partial differential equations via Fourier methods, concepts from linear algebra and ordinary differential equations.

[4] You may recall that these values are determined by the initial conditions, eigenvalues and eigenvectors, respectively.

[5] In the previous problem the solution space has a two-dimensional basis and thus all solutions to the problem can be written using linear combinations of these two eigenvectors. This is analygous to the concept that any vector in the plane can be written as the linear combination of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

[6] We won't question it here and for many the theory is not needed for the fearless calculations. However, we should at least make a mental note that we will be on slippery slope. One can show that there is a convergent trigonometric series that is not the Fourier series of any integrable function and at some point we will construct a series used to represent a function, which is allowed be called equivalent even though we permit it to differ from the function at a countably-infinite amount of points, bringing into question what we actually mean by *integral*.

MATH348 - Spring2009 - Tentative Schedule

Lecture(s)	**Section**	**Pages**	**Key Concepts**
1	7.1,7.2	272-286	Algebra, Associativity, Commutativity, Distribution, Inner-Product, Outer-Product, Matrix Product, Symmetric, Skew-Symmetric
2-4	7.3,7.5	287-295, 302-305	Linear System, Existence and Uniqueness, Gauss Elimination, Row Echelon Form, Fundamental Theorem for Linear Systems, Homogeneous and Nonhomogeneous systems.
6-7	7.7-7.8	308-314	Determinant, Cramer's Theorem, Matrix Inverse, Orthogonal Matrix
8-10	7.4, 7.9	296-301, 323-329	Linear Dependence, Basis, Dimension, Rank, Span, Row Space, Column Space, Null Space, Vector Space, Inner Product Space
11	8.1	334-339	Eigenvalue, Spectra, Eigenvector, Eigenfunction
12	8.3	345-348	Symmetric, Skew-Symmetric, Orthogonal, Transformations, Spectra
13-14	8.4	349-355	Eigenbasis, Diagonalization, Quadratic Form, Definiteness
15-16	Review of Functions	N/A	Function, Even, Odd, Periodic Function, Trigonometric Function, Factorial Function, Gamma Function, Bessel Function of the First Kind
17-18	11.1, 11.3	478-486, 490-495	Fourier Series, Fourier Coefficents, Fourier Series of Functions with Symmetry
18	11.2	487-489	Domain Scaling Properties
19	11.4	496-498	Euler's Formula, Complex Fourier Series
20	11.6	502-505	Trigonometric Approximation
21	11.7-11.8	506-517	Fourier Integral, Fourier Sine/Cosine Transform
22-25	11.9	518-528	Fourier Transform, time/space domain, frequency domain, spectral representation, convolution, Green's function, Frequency Response
26	Review of DE, 12.1	535-537	Differential Equation, Vocabulary, Linear ODE's, Boundary Value Problems, Simple Harmonic Oscillators, Bessel's Equation
27-28	Flows and Conservations Laws	N/A	Divergence Theorem, Conservation Equation, Constitutive Equation, Fourier's Law of Heat Conduction
29	12.5	552-561	Boundary Conditions, Separation of Variables, Periodic Extension
30	Inhomogeneity	N/A	Extension of Fourier Methods
31	12.2-12.4	538-551	Ideal Wave Equation, Vibrations, D'Alebert's Solution
32	12.6	562-568	Cauchy-Problem, Heat Kernel
33	12.9	579-586	Multivariate Chain Rule, Laplacian in Polar Coordinates, Fourier-Bessel Series
34	12.10	587-593	Cylindrical and Spherical Geometries
35	12.11	594-596	Laplace Transforms and PDE's
36	Acoustics	N/A	Linear Approximations and Small Amplitude Vibrations