Note Title 9/27/2006

9/27/06

Exam wed. oct 11

monday @ 2 Roel Snieder
Has office Hours.

Available at some time
tuespay. 12:00 ? 8:00 ?

I'LL be @ OSA

No. class monday 10/9
special lecture on 10/13

9/27/06

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$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \xrightarrow{\text{row red.}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

# of lin. ind. rows =
lin. ind columns = 2

Null space?

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} 2x + 3y &= 0 \\ x + 2y &= 0 \\ x + 3y &= 0 \end{aligned}$$

equivalent to

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \quad x = 0, \quad y = 0$$

NO NON-trivial Null space.

Hence solution must be unique

Augmented matrix

$$\left[\begin{array}{cc:c} 2 & 3 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 5 \end{array}\right] \longrightarrow \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

new (equivalent) system of equations

$$\boxed{\begin{aligned} x &= -4 \\ y &= 3 \end{aligned}}$$

original system was

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

so $\boxed{-4 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}}$

Example 2.

$$x + 2y - z = 1$$
$$2x + 3y - 2z = -1$$
$$3x + 4y - 3z = -4$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -2 \\ 3 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$$

$\downarrow$ row reduces to

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

rank = 2

null space = ?

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $x - z = 0$
$y = 0$

Null space spanned by
$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = N(A)$$

we can add this to ANY solution and get another solution.. That is, if there is a solution.

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 3 & -2 & -1 \\ 3 & 4 & -3 & -4 \end{array}\right]$$

$\downarrow$ row reduce

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

what does this mean?

suppose this were a parameter α.

$\Rightarrow$ $x - z = 0 \Rightarrow x = z$

$y = 0 \Rightarrow y = 0$

$0 = \alpha$ True $\Leftrightarrow \alpha = 0$

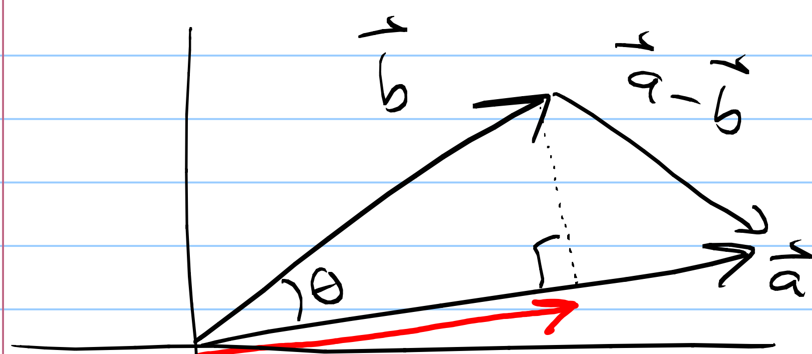
So unless $\alpha = 0$, RHS not in column space of A.

This means there is no ordinary solution.

Mathematica Digression

Projection operators

crucial for QM (320)



$\|\vec{b}\| \cos\theta$ = length

$\hat{a}$ = direction

$$\hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$$

$$\vec{b} \cdot \vec{a} = \|\vec{b}\| \, \|\vec{a}\| \cos\theta$$

$\hat{a}$

$$\|\vec{b}\| \cos\theta \; \hat{a} = \underbrace{\frac{\vec{b}\cdot\vec{a}}{\|\vec{a}\|}}_{\text{scalar}} \frac{\vec{a}}{\|\vec{a}\|}$$

$$= \frac{\vec{a}}{\|\vec{a}\|} \frac{\vec{b}\cdot\vec{a}}{\|\vec{a}\|}$$

$$\boxed{\vec{x}\cdot\vec{y} = \vec{y}\cdot\vec{x}}$$

$$= \frac{\vec{a}}{\|\vec{a}\|} \frac{\vec{a}\cdot\vec{b}}{\|a\|}$$

$$= \frac{\vec{a}\,\vec{a}^T}{\|\vec{a}\|^2} \cdot \vec{b} = \frac{\vec{a}\,\vec{a}^T}{\vec{a}^T\vec{a}} \cdot \vec{b}$$

$$\boxed{\frac{\vec{a}\,\vec{a}^T}{\vec{a}^T\vec{a}} \quad \text{projection operator}}$$

outer product of 2 vectors is

outer $\quad (\vec{x}\,\vec{y}^T)_{ij} = x_i\, y_j$

Inner prod. $\quad \vec{x}^T \vec{y} = \sum_{i=1}^{N} x_i\, y_i$

~ Dyadic product

Projection of a vector

onto $\vec{a}$

matrix

$$\frac{\vec{a}\,\vec{a}^T}{\underbrace{\vec{a}^T\vec{a}}_{\text{scalar}}}$$

$$P \equiv \frac{\vec{a}\,\vec{a}^T}{\vec{a}^T\vec{a}}$$

Deriving the <u>normal equations</u>

1) $Ax - y = r \neq 0$

r not in Range(A) = column space. Hence must be in Nullspace of A^T.

$$\Rightarrow \quad A^T(Ax - y) = A^T r = 0$$

$$\boxed{A^T A x = A^T y}$$