Generalizations for non-relativistic frames $\Rightarrow$ relativistic frames.

Proper time: $d\tau = \sqrt{1-u^2/c^2}\, dt = \frac{1}{\gamma} dt$

$d\tau$ ↑ moving object dt

dt ↑ laboratory dt

u = speed of object w/ respect to laboratory.

Proper velocity: $\vec{\eta} = \gamma \vec{u} = \frac{d\vec{l}}{d\tau}$ $\left\{ \vec{u} = \frac{d\vec{l}}{dt} \right\}$

$d\vec{l}$ ← lab frame

$d\tau$ ← object frame.

$\vec{u}$ can't be generalized as a 4-vector.

But for $\vec{\eta}$, you can define a contravariant vector $\eta^\mu = \begin{pmatrix} \gamma c \\ \gamma \vec{u} \end{pmatrix}$

$\Rightarrow \bar{\eta}^\mu = \frac{d\bar{x}^\mu}{dx^\nu} \eta^\nu$

Momentum: $\vec{p} = \gamma m \vec{u} = m\vec{\eta}$

$p^\mu = m\eta^\mu = \begin{pmatrix} \gamma m c \\ \gamma m \vec{u} \end{pmatrix}$ ← $\frac{\text{relativistic energy of obj.}}{c}$

$E = \gamma m c^2$

$E_{kin} = E - E_{rest} = (\gamma - 1) m c^2$

$= \left(\frac{1}{\sqrt{1-u^2/c^2}} - 1 \right) m c^2$

$= \frac{1}{2} m u^2 + \frac{3}{8} \frac{m u^4}{c^2} + \cdots$

$\Rightarrow$ Find $p_\mu p^\mu \ldots -\frac{E^2}{c^2} + p^2 = ?$

$E = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} m c^2$ $\quad \vec{p} = \frac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}}$

$$\Rightarrow \underbrace{-\frac{E^2}{c^2} + p^2 = -m^2 c^2}$$

Back in Modern: $E^2 - (pc)^2 = (mc^2)^2$

See last page.

Newton's Laws: $\vec{F} = \frac{d\vec{p}}{dt}$ ✓ $\vec{p}$ is

where $\vec{F} = q\vec{E} + q\vec{u} \times \vec{B}$.

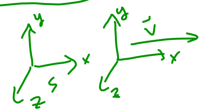
$\vec{F}$ can't be generalized as a 4-vector

We define $\vec{K} = \frac{d\vec{p}}{d\tau}$ {Minkowski Force}

$$K^\mu = \frac{dp^\mu}{d\tau} \Rightarrow K^0 = \frac{1}{c}\frac{dE}{d\tau}$$

$$K_\mu K^\mu = \frac{1 - u^2/c^2 \cos^2\theta}{1 - u^2/c^2} F^2$$

Now for E & M:



$$\bar{E}_x = E_x \qquad \bar{E}_y = \gamma(E_y - vB_z) \qquad \bar{E}_z = \gamma(E_z + vB_y)$$

$$\bar{B}_x = B_x \qquad \bar{B}_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right) \qquad \bar{B}_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

$$\bar{F}^{\alpha\beta} = \frac{d\bar{x}^\alpha}{dx^\mu}\frac{d\bar{x}^\beta}{dx^\nu} F^{\mu\nu} = \Lambda^\alpha_\mu \Lambda^\beta_\nu F^{\mu\nu}$$

$\frac{\partial}{\partial x^\nu}$ sometimes is written as ∂_ν and is covarient.

$\frac{\partial}{\partial x_\nu}$ is written ∂^ν and is contravariant.

These are the 4-vector generalization of $\vec{\nabla}$. $\frac{\partial F^{\mu\nu}}{\partial x^\mu}$ is like $\vec{\nabla} \cdot \overleftrightarrow{F}$

Also, you can define

$$G^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -\frac{E_z}{c} & E_y/c \\ -B_y & E_z/c & 0 & -E_x/c \\ -B_z & -E_y/c & \frac{E_x}{c} & 0 \end{pmatrix}$$

Current & charge:

$$J^\mu = (c\rho, \vec{J})$$

Continuity eqn.

$$\frac{\partial J^\mu}{\partial x^\mu} = \emptyset$$

Maxwell's eqns become.

$$\boxed{\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu \qquad \frac{\partial G^{\mu\nu}}{\partial x^\nu} = \emptyset}$$

Potentials become

$$A^\mu = (V/c, \vec{A})$$

$$F^{\mu\nu} = \frac{\partial A^\nu}{\partial x_\mu} - \frac{\partial A^\mu}{\partial x_\nu}$$

Really in order for A^μ to be a 4-vector (contravarient), not all gauges work. The Lorentz gauge works.

Maxwell's eqns in terms of potentials is

$$\boxed{\partial^\mu \partial_\nu A^\nu = \mu_0 J^\mu}$$

$$\frac{\partial}{\partial x_\mu} \frac{\partial}{\partial x^\nu} A^\nu = \mu J^\mu$$