

Phase-matching in cascaded third-order frequency mixing

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Outline

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- Applications of ultrafast shortwavelength pulses
- Background: alternatives for frequency conversion
- Techniques for hollow-core waveguide phase matching
- Phase-matching of cascaded mixing
 - analytic phase-matching conditions
 - simulations
 - experimental results
- Scaling to greater conversion and higher orders

Applications of ultrafast short-wavelength pulses



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- Chemical reactions

- > time resolution
- > intermediate states
- > coherent control

- Materials

- > micromachining/nanofabrication
- > lithography
- > synchrotron applications

- Physics

- > atomic stabilization
- > high density plasma interactions
- > x-ray nonlinear optics

- Biophysics

- > cross-linking DNA/proteins
- > water-window holography



Frequency conversion with nonlinear crystals

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Limitations of nonlinear crystals:

- Transparency ~ 150-200 nm
- Phase-matching range ~ 190 nm
- Phase-matching bandwidth/
group velocity walkoff ~ 150 fs @266nm
- Periodically-poled materials ~ 350 nm
- Crystal damage for high-power pulses



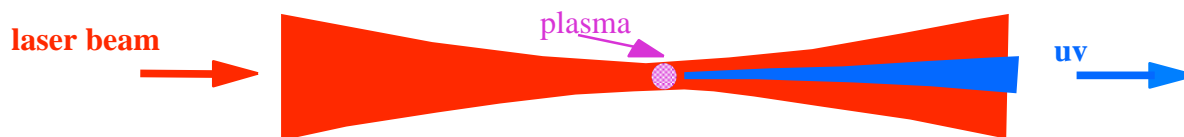
Frequency conversion in gases

Use of gases circumvents some of these problems:

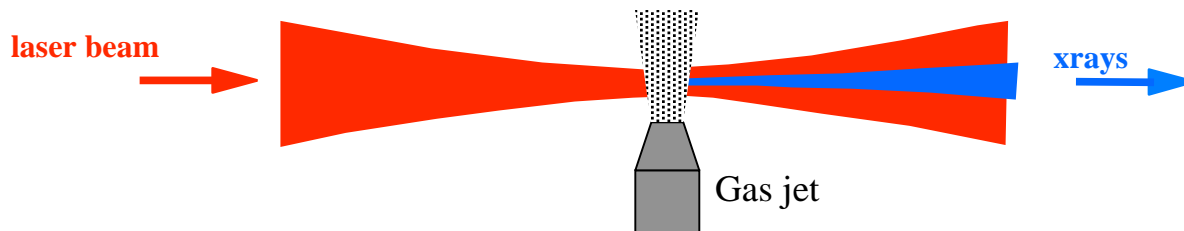
- High transparency: He: >50 nm, Ar: >105 nm
- Low dispersion preserves pulse width
- No damage limitations

Focused beam geometry: Guoy phase shift

- direct third-harmonic
($1 \mu\text{J}$, 0.1% , 12 fs)



- High-order harmonic generation





Hollow waveguides for high-intensity guiding

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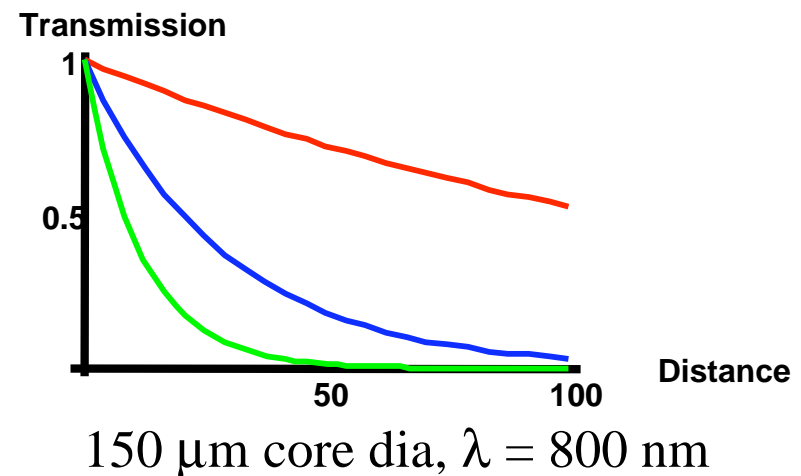
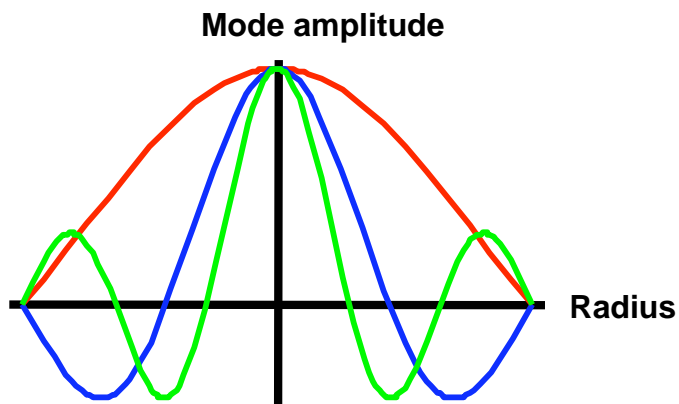
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Low conversion efficiency in gases:

- Improve $intensity \times density \times length$
- Phase-matching

Capillary waveguides:

- guiding is lossy but allows high-intensity interactions

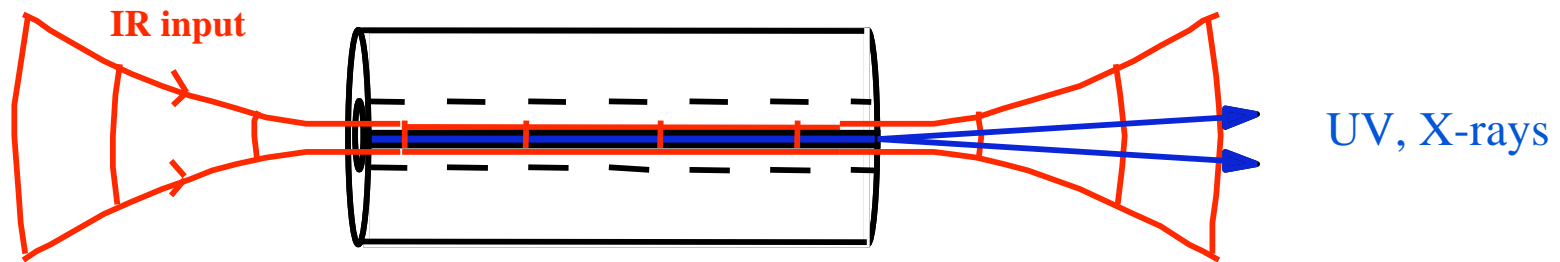


Hollow waveguide propagation phase



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The waveguide modes have flat wavefronts → z-independent contribution to phase.

$$k = \frac{2\pi}{\lambda} \left[1 + P\delta(\lambda) - \frac{1}{2} \left(\frac{u_{nm}\lambda}{2\pi a} \right)^2 \right]$$

vacuum gas waveguide

Waveguide phase from countering diffraction: strong effect on long wavelength

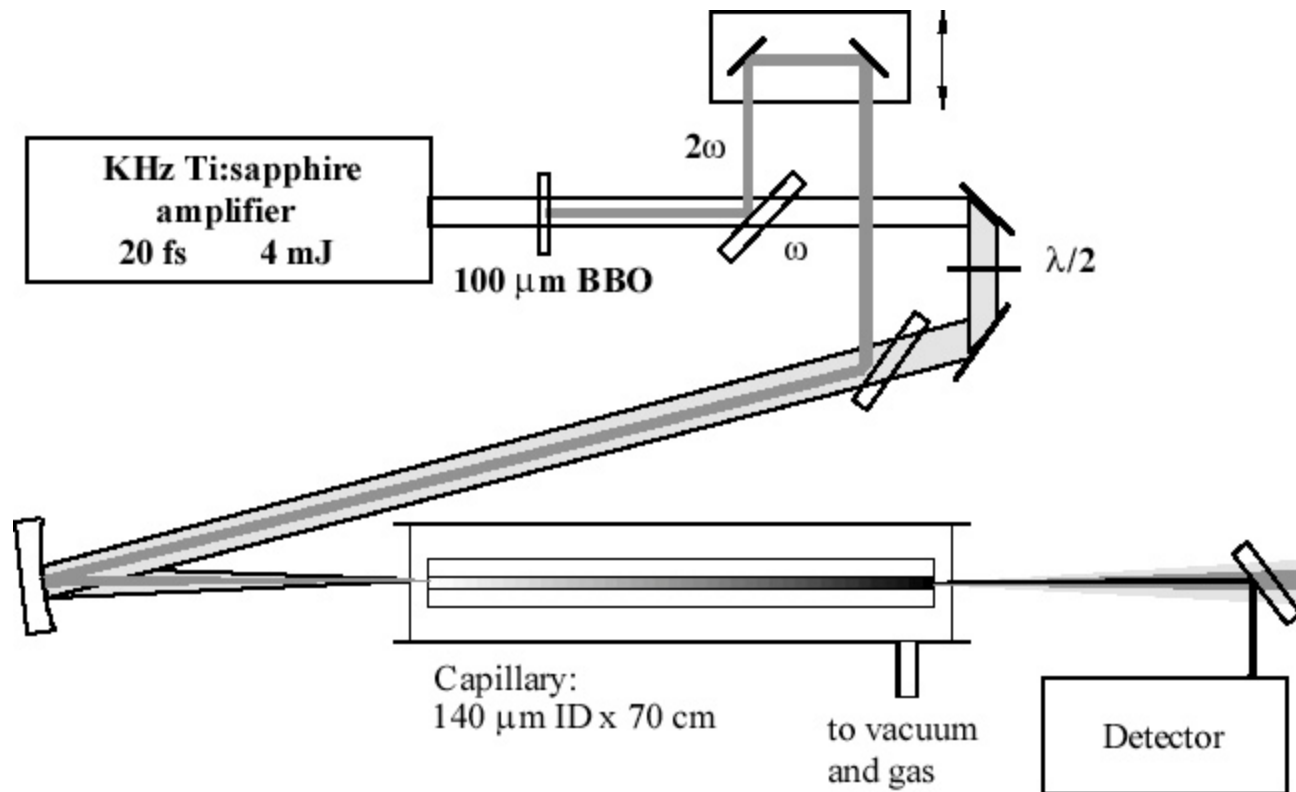
Phase match ($\Delta k = 0$) by varying gas pressure to balance gas and waveguide dispersion.

Experimental setup



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Capillary lengths vary or can be segmented



Techniques for waveguide phase matching

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Modal phase matching for direct harmonic generation

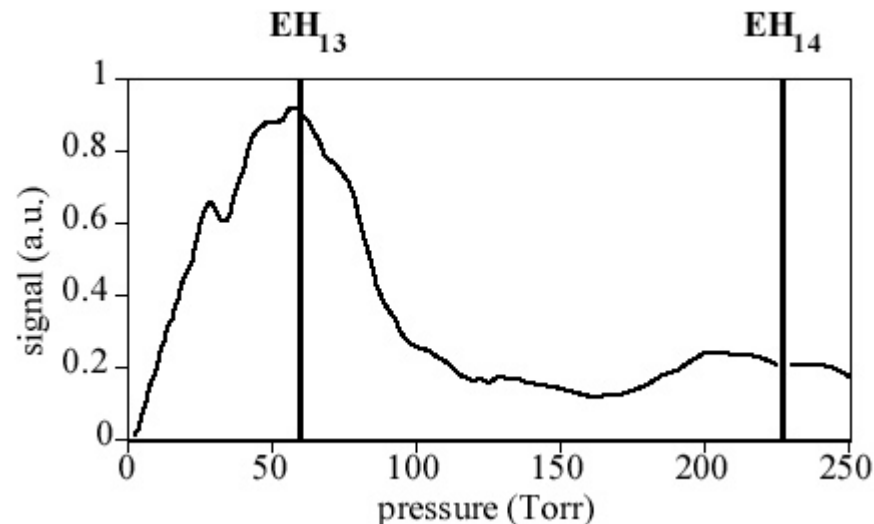
(Durfee et al, Opt. Lett. 22 p. 1565 (1997))

$$\Delta k = \frac{2\pi P}{\lambda_3} [\delta(\lambda_3) - \delta(\lambda_1)] - \frac{1}{4\pi a^2} (u_{13}^2 \lambda_3 - 3u_{11}^2 \lambda_1)$$

$$\Delta k_{material} > 0$$

$$\Delta k_{mode} > 0$$

Experimental
data: Argon



Efficiency is low ($\sim 0.2\%$) because of poor mode overlap



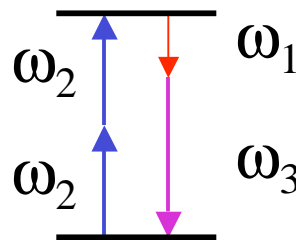
Techniques for waveguide phase matching

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Frequency mixing allows all beams to be in lowest mode

$$\omega_3 = 2\omega_2 - \omega_1$$



$\omega_1 =$ idler

$\omega_2 =$ pump

$\omega_3 =$
signal

$$\Delta k = 2\pi P \left[\frac{\delta(\lambda_3)}{\lambda_3} - \frac{2\delta(\lambda_2)}{\lambda_2} + \frac{\delta(\lambda_1)}{\lambda_1} \right] - \frac{u_{11}^2}{4\pi a^2} (\lambda_3 - 2\lambda_2 + \lambda_1)$$

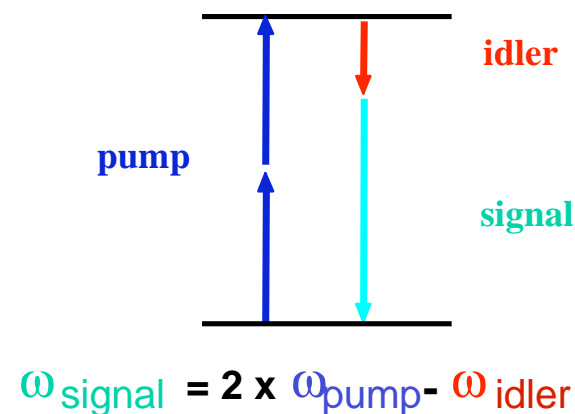
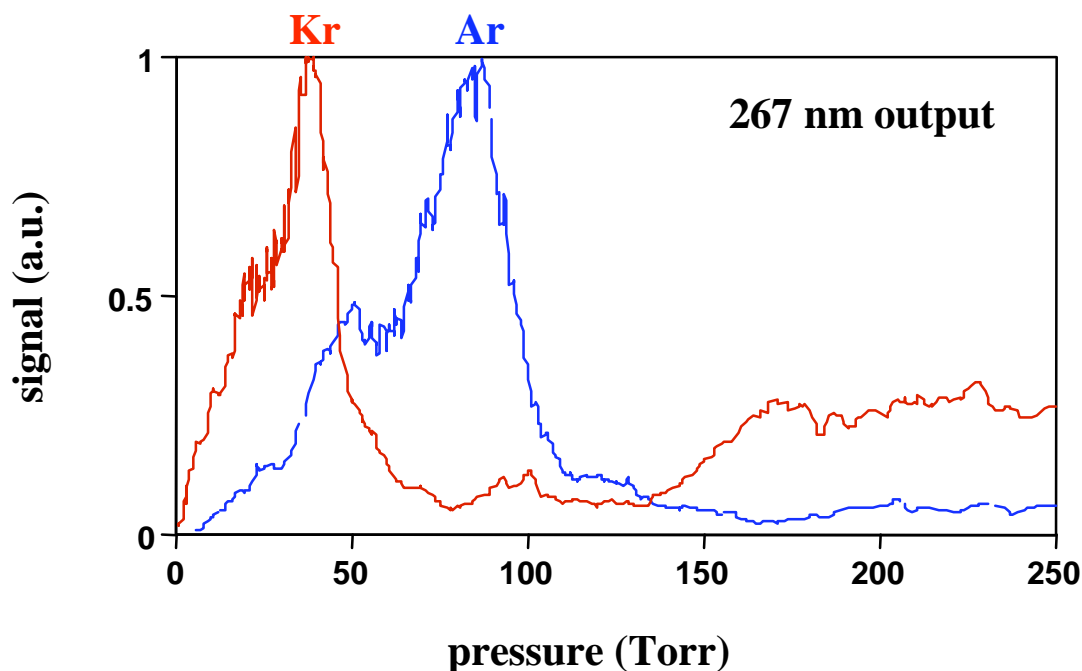
Phase-matching: Tune pressure to balance gas and modal terms



Waveguide phase matching: UV results

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~ 40% conversion from blue pump
near-Gaussian output mode
< 7 fs walkoff

Tunable throughout the deep-UV
by using OPA output as idler

Durfee et al, Opt. Lett. 22 p. 1565 (1997)

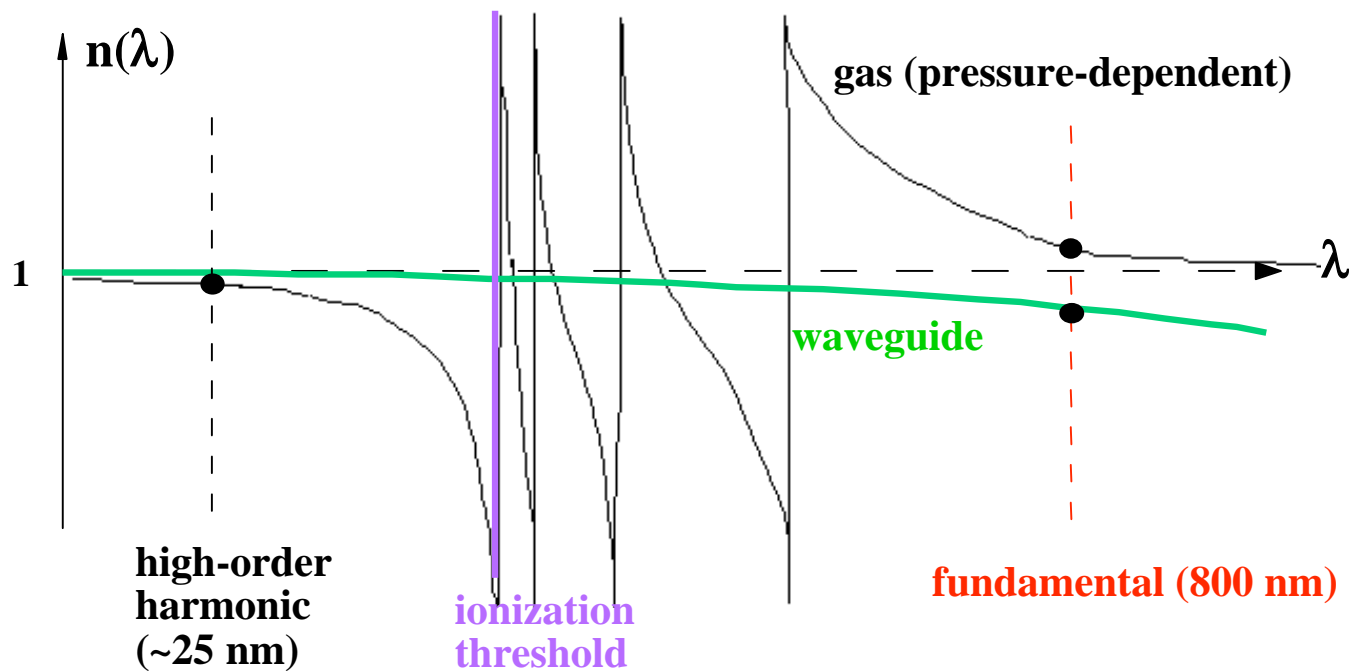
Which gas? *High nonlinearity=high dispersion=lower pressure.*
Doesn't really matter.



Waveguide phase matching: high harmonics

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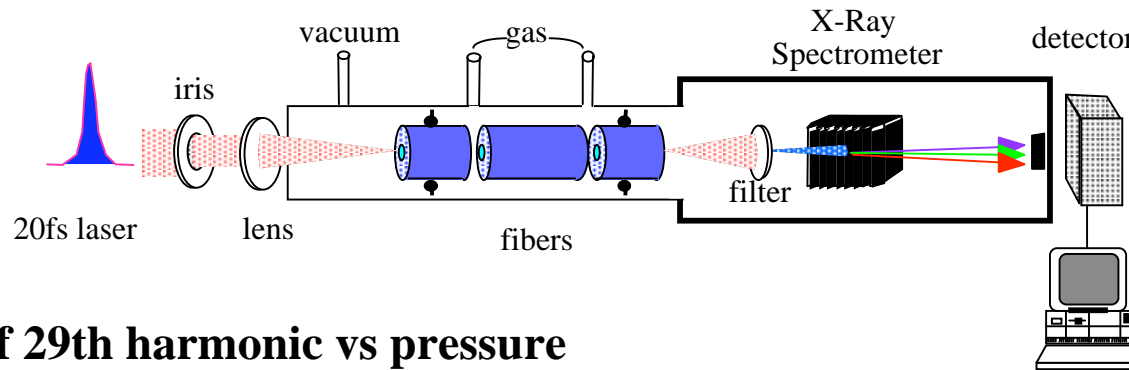
- ♦ Gas, waveguide dispersion balance.
- ♦ $n < 1$ @ 25 nm -> Simple HHG can be phase-matched.
- ♦ Absorption of signal is now an issue

Phase-matched HHG results

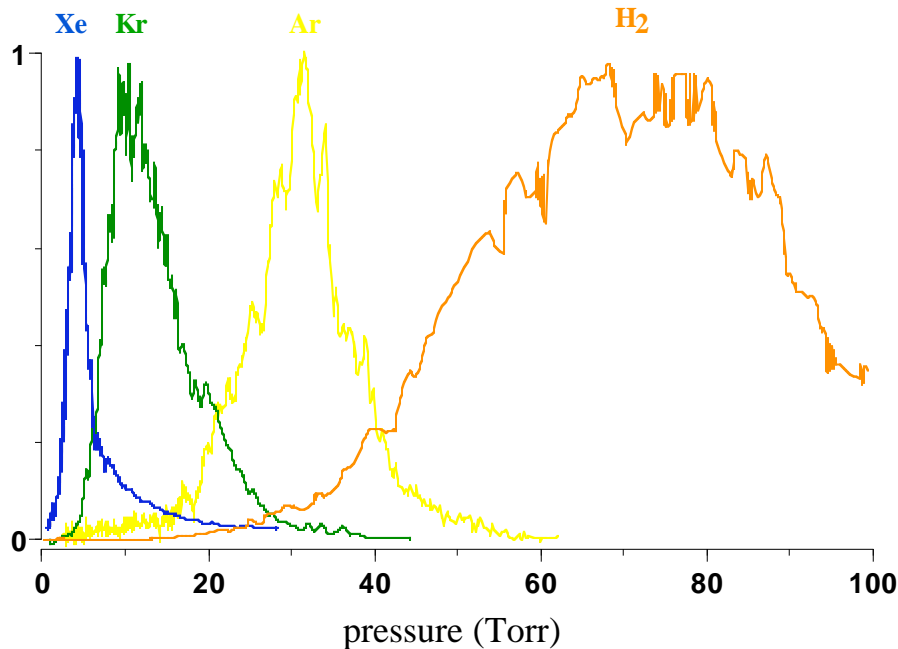


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Yield of 29th harmonic vs pressure



- Output enhanced by 100-1000x
- Yield limited by absorption
- optimum pressure varies with gas
- no defocusing, little absorption of fundamental

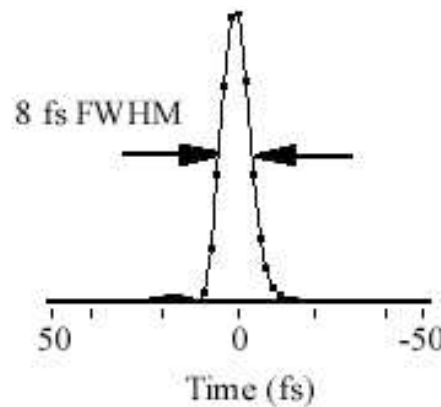
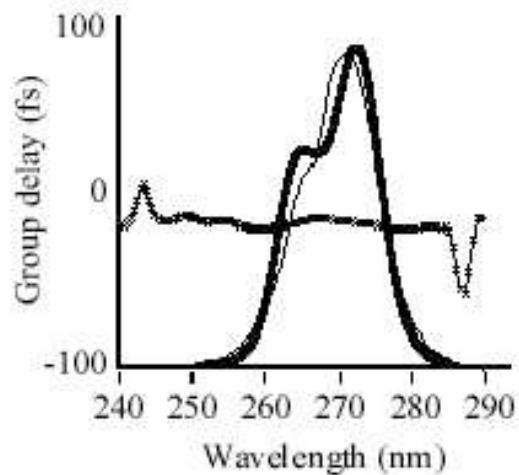
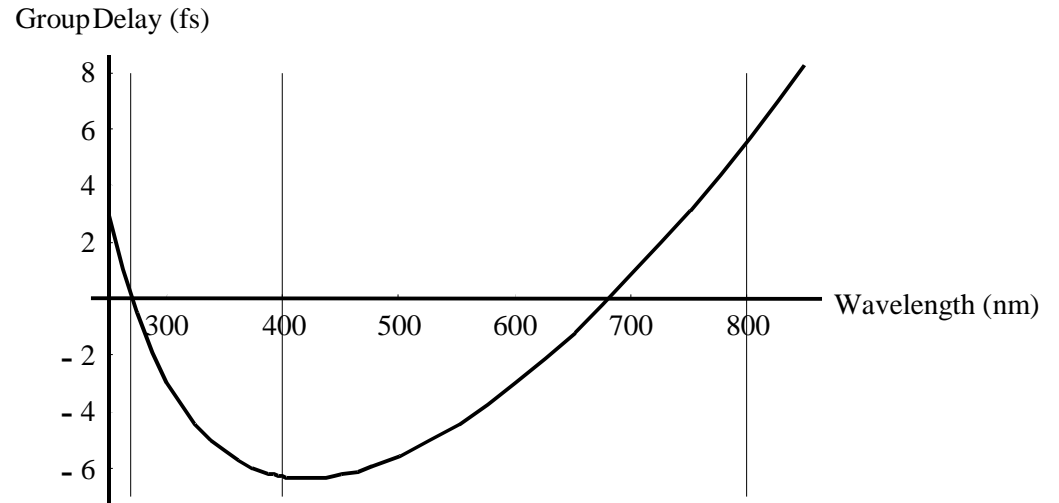
Rundquist *et al*, Science **280**, 1412 (1998)
Durfee *et al*, PRL **83**, 2187 (1999)

Short UV pulse generation/compression



High pulse intensity: SPM, XPM

Low degree of walkoff allows short pulse generation



Post-compression with gratings yields ~ 8 fs, μ J pulses



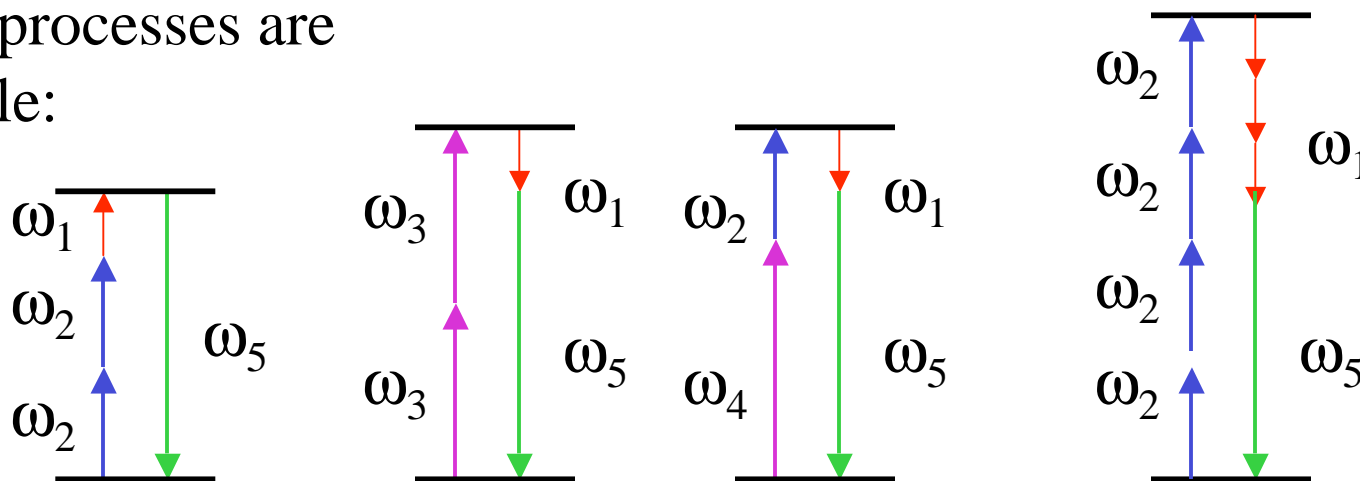
Cascaded processes

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High conversion to third harmonic drives cascaded processes.

Many processes are possible:



Which paths are most important?

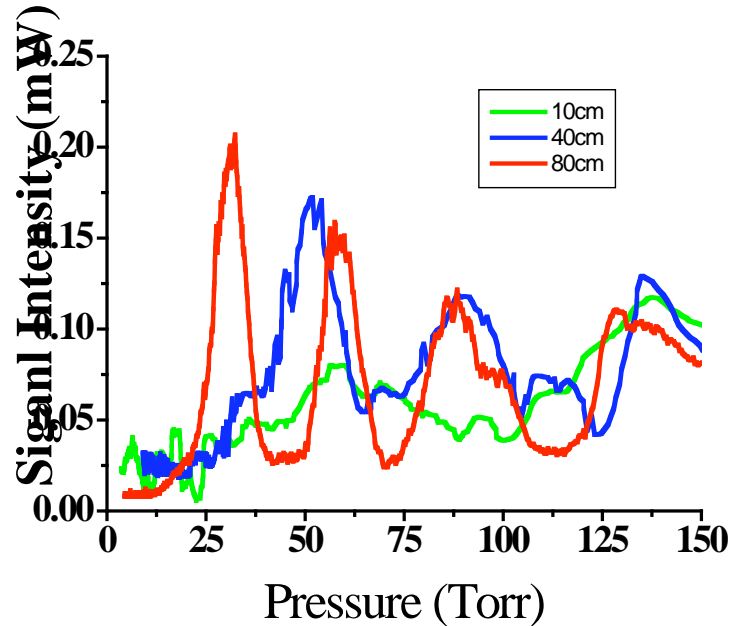
- phase-matching
- effective order of nonlinearity

Cascaded generation: experimental results

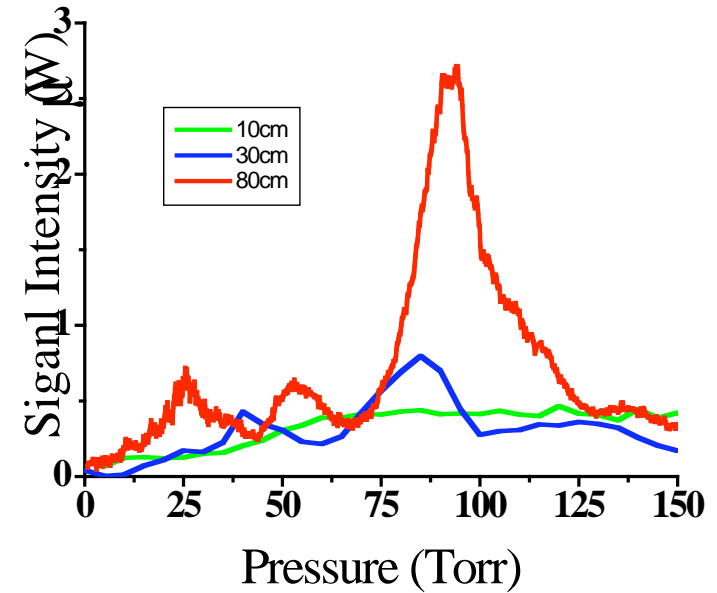


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$$\omega_4 = 200\text{nm} (\sim 1\%)$$



$$\omega_5 = 160\text{nm} (\sim 0.01\%)$$

Cascaded conversion is relatively efficient, shows distinct signature of phase matching.



Approaches to calculations

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Two approaches to calculations:

1. Analytic - small-signal analysis

- phase-matching conditions
- estimate relative efficiency of different processes

2. Numerical modeling

- saturation
- walkoff
- phase-modulation effects
- losses
- higher-order modes



Small-signal analysis: phase-matching

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To derive phase-matching conditions:
use non-depleted growth of third harmonic as source for cascading

Example:

$$\frac{dE_4}{dz} = ik_4 \chi^{(3)} E_2 E_3(z) E_1^* \exp[i\Delta k_4 z]$$

Where

$$E_3(z) = \frac{-2k_3 \chi^{(3)} E_2^2 E_1^* \exp[i\Delta k_3 z / 2] \sin[\Delta k_3 z / 2]}{\Delta k_3}$$

E_4 grows under several different phase matching conditions...

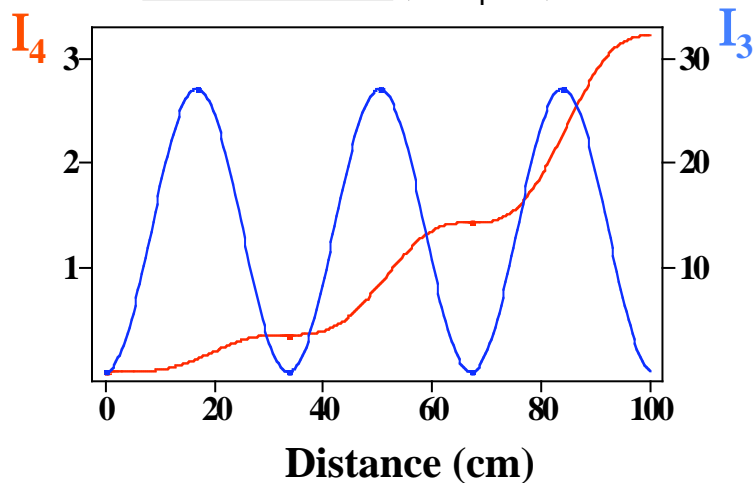
Cascaded phase-matching mechanisms



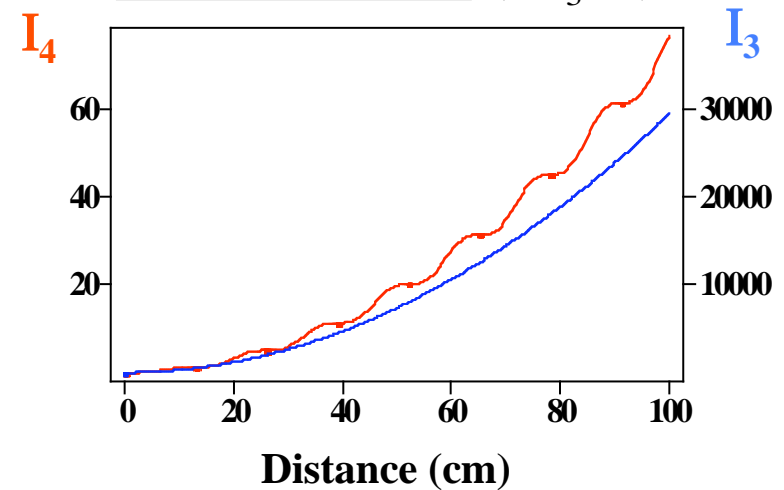
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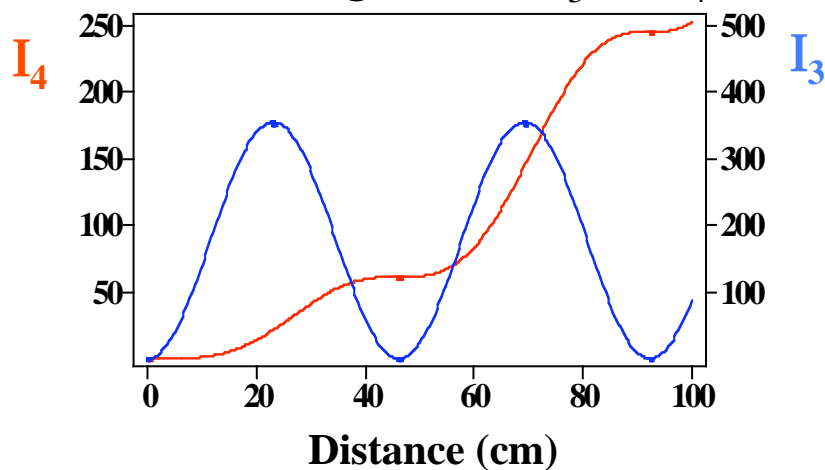
Standard ($\Delta k_4=0$)



Gain-assisted ($\Delta k_3=0$)



Cascaded QPM ($\Delta k_3 + \Delta k_4 = 0$)



Gain-assisted PM:

Increase in $E_3(z)$ gives buildup in E_4 in spite of phase mismatch

Cascaded QPM:

P_{NL} is modulated by $E_3(z)$ @ $1/\Delta k_4$

Cascaded vs. direct processes



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For sufficiently long interaction length, cascaded low-order processes can dominate direct high-order.

Effective nonlinearity for cascading:

$$\chi_{eff}^{(5)} \approx \frac{2\pi k_3 (\chi^{(3)})^2}{\Delta k_3}$$

Residual phase mismatch (Δk_3) is *small* in gases:

Example: for $\omega_4 = \omega_2 + \omega_3 - \omega_1$ and $\Delta k_4 + \Delta k_3 = 0$

$$I_{direct}/I_{cascaded} \sim 10^{-5}$$

Nonperturbative effects:

- χ does not drop off as steeply: plateau in high orders >7

Propagation calculations



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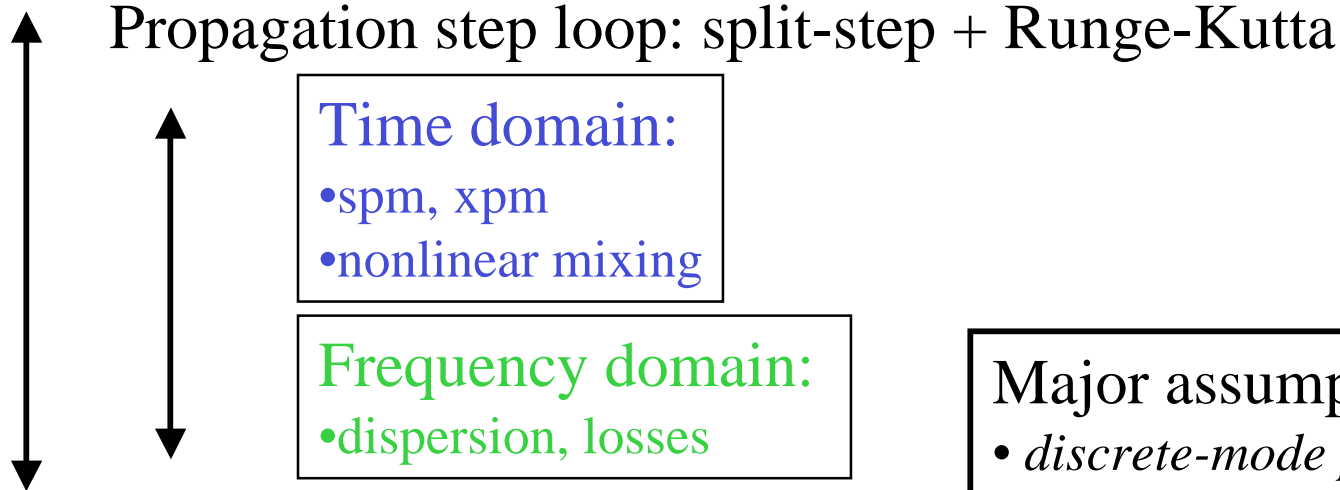
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Propagation code calculates saturated conversion

Input fields:

- energy, pulse duration, chirp, relative delay

Pressure loop



Time domain:

- spm, xpm
- nonlinear mixing

Frequency domain:

- dispersion, losses

Output processing:

- energy calculation
- post-compression

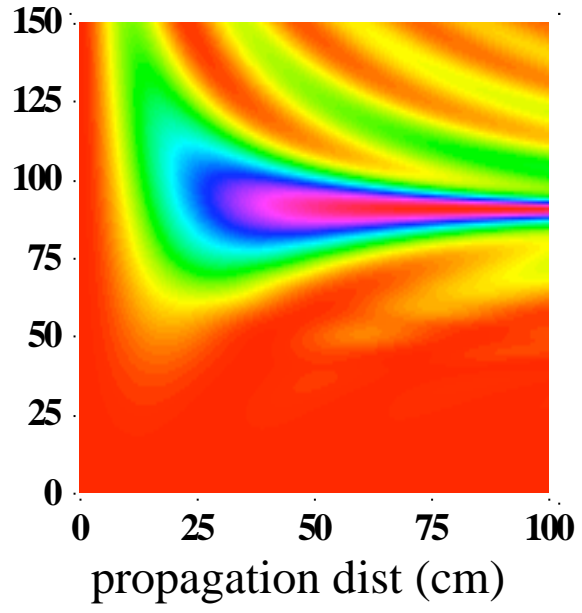
Major assumptions:

- *discrete-mode propagation*
- *five harmonic fields*
- *no bending losses*
- *no ionization*

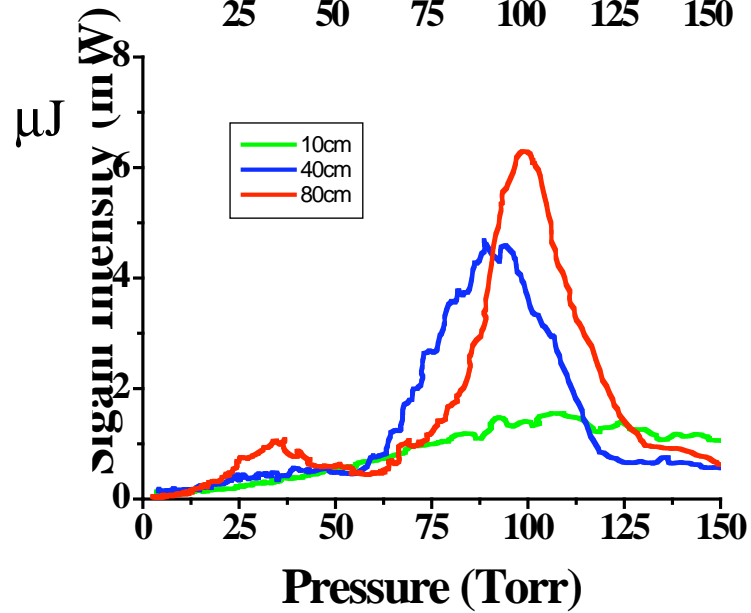
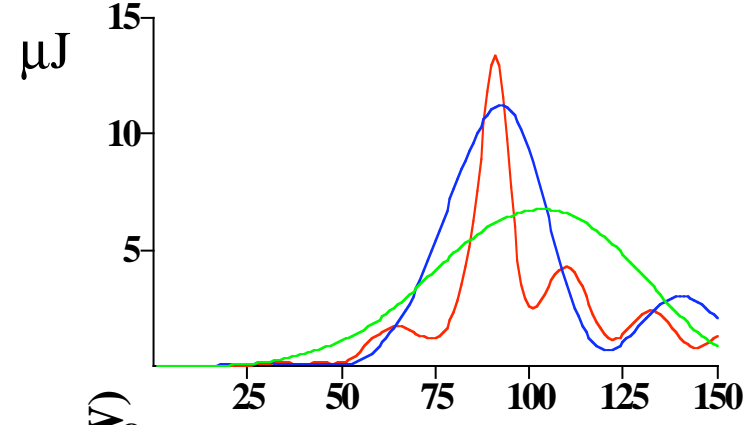
Code results: third harmonic



P (torr)



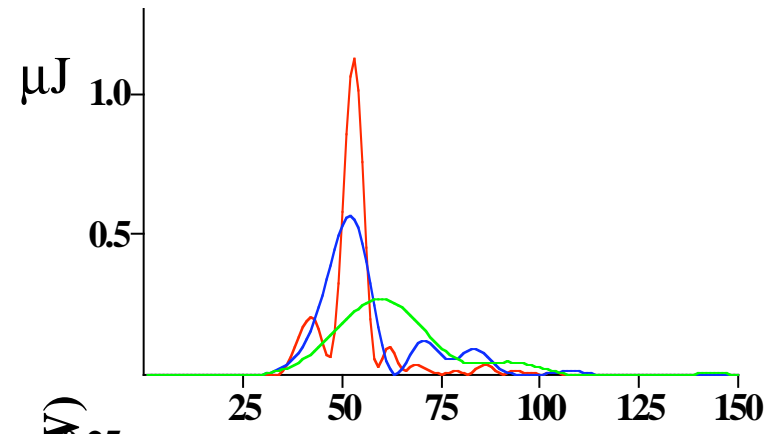
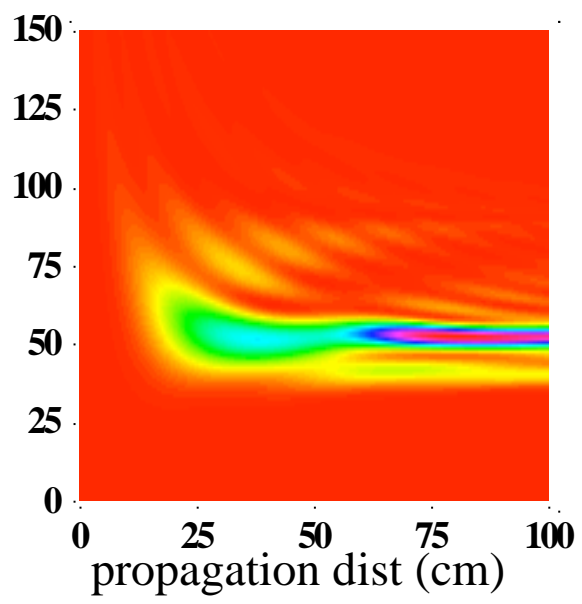
Low pressure:
peak broad, shifted high



Code results: Fourth harmonic



P (torr)

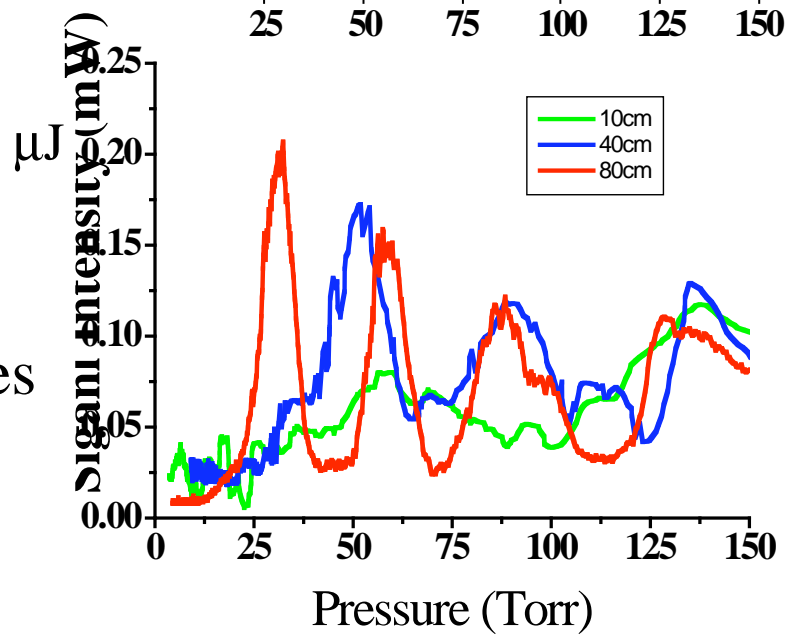


Code:

- cascaded QPM peak dominates

Measurement:

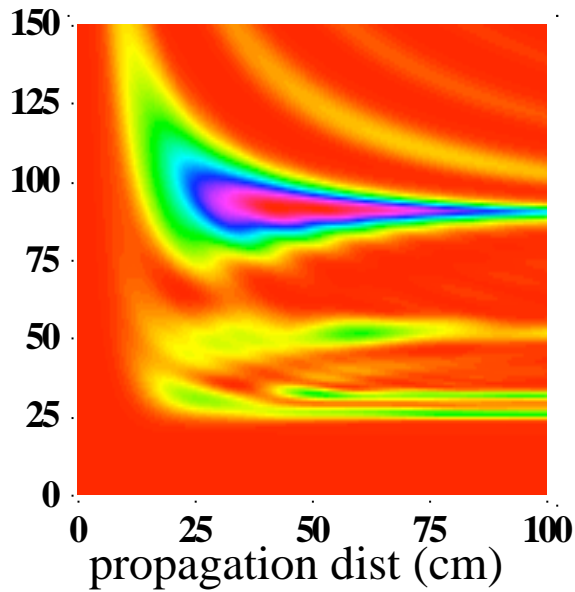
- all three important



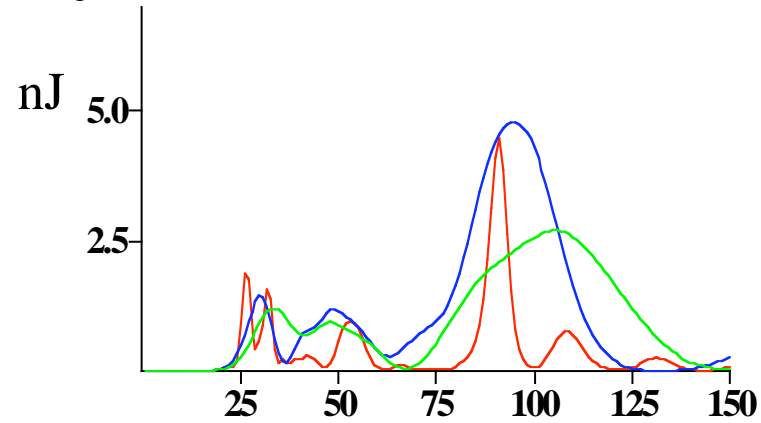
Code results: fifth harmonic



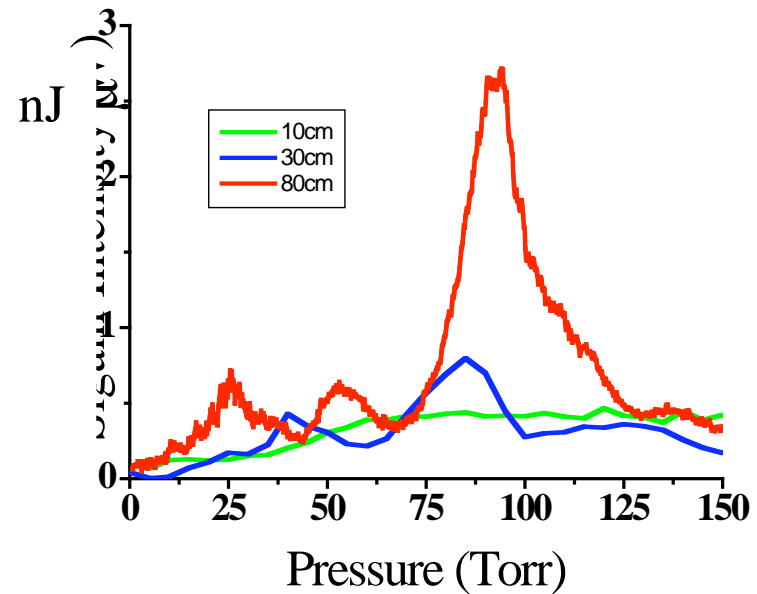
P (torr)



E_{sig} (nJ)



Gain-assisted peak is strongest

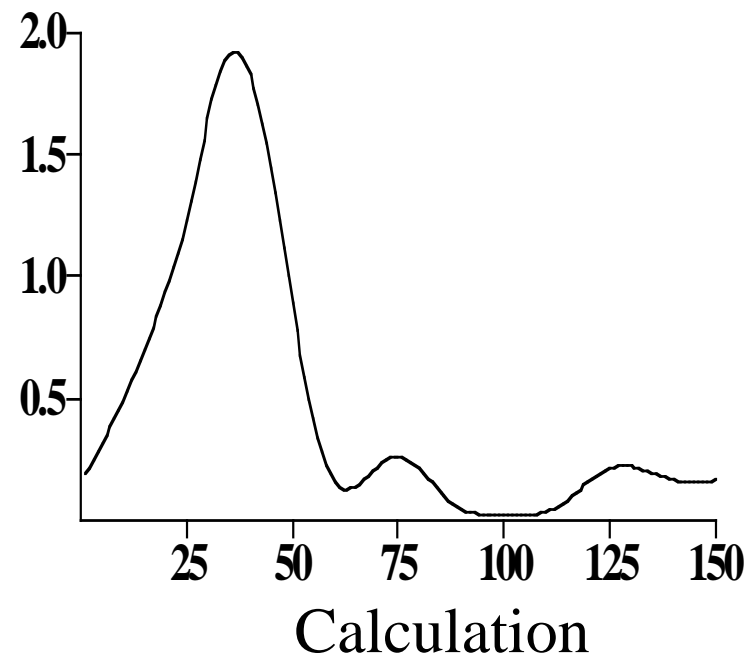
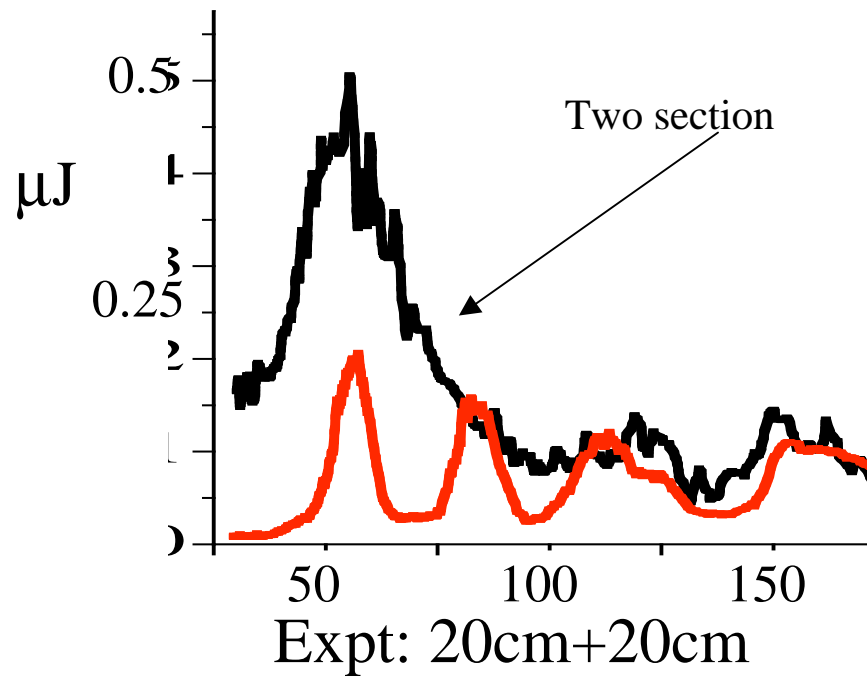




Two-stage generation

Improve cascading efficiency by using two stages:

1. Generate w_3 : $\Delta k_3=0$
2. Generate w_4 : $\Delta k_4=0$ (not CQPM)



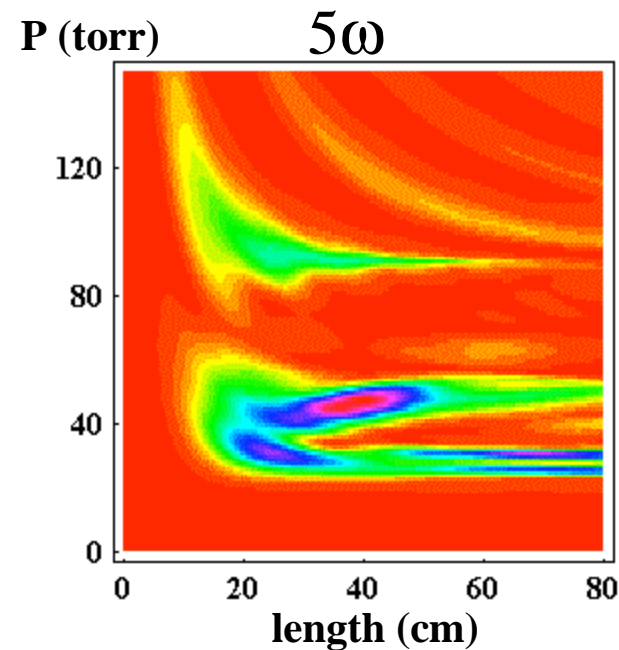
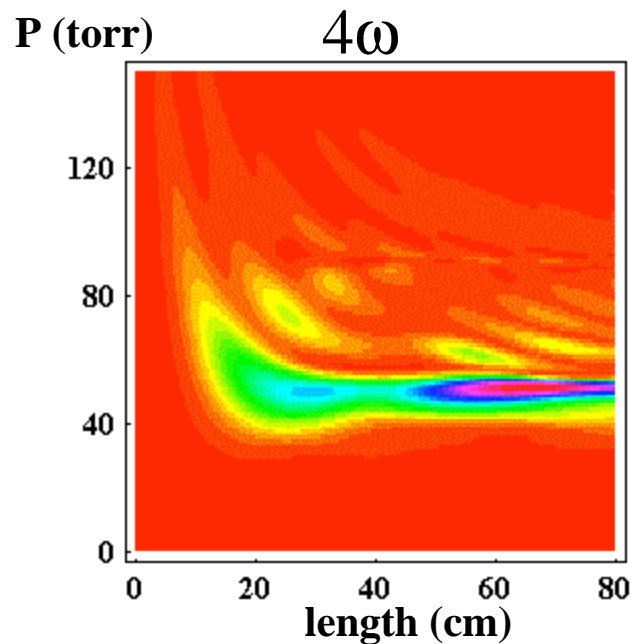
Scaling to high conversion



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High intensity blue (400nm) drives conversion to 4ω and 5ω
- should approach $\sim 20\%$, 2% conversion



At $150\mu\text{J}$ (400nm), $10\mu\text{J}$ (800nm), optimum 5ω shifts to peak 4ω output (gain-assisted).

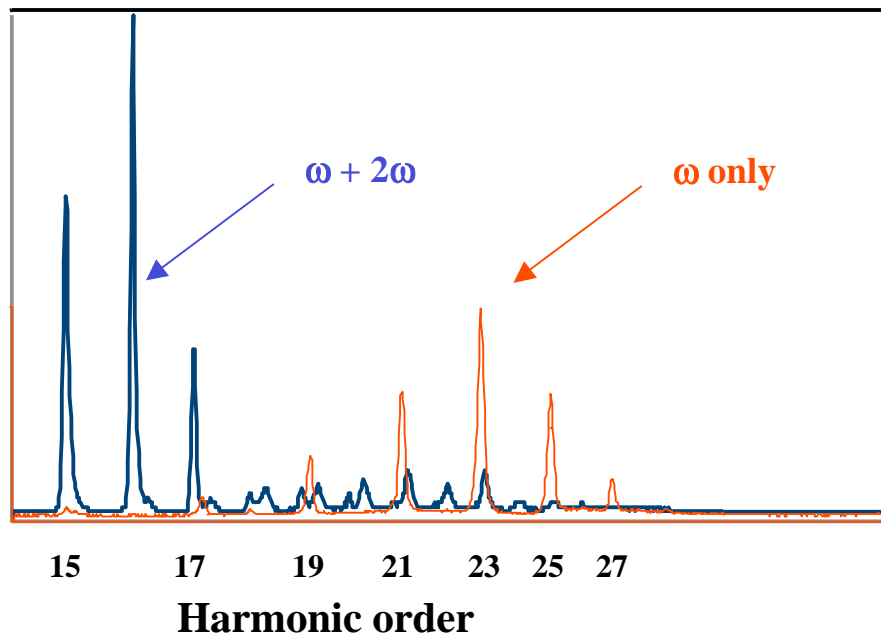
High-order cascading



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Harmonic yield



3 cm capillary
150 μ J @ ω
90 μ J @ 2ω

Mixing ω and 2ω gives significant enhancement of HHG
inside absorption window



Conclusions and future work

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Summary:

- efficient upconversion to deep-UV, VUV, and XUV
- three types of phase-matching:
 - standard
 - cascaded quasi-phase-matching
 - gain-assisted phase-matching

Future work:

- improve efficiency of cascading
- extend to higher orders
- scale to higher pulse energies
- develop high power fs sources at 400nm and UV wavelengths