

only get V along z axis. Now find $\vec{E} = -\vec{\nabla} V$

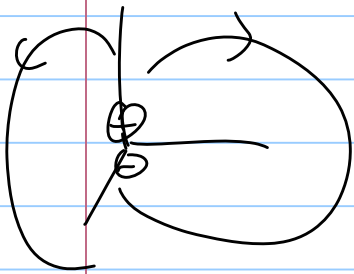
$$= -\frac{\partial V}{\partial x} \hat{x} - \frac{\partial V}{\partial y} \hat{y} - \frac{\partial V}{\partial z} \hat{z}$$

find V only on z axis

$$V(z) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}') d\vec{r}'}{r}$$

expand $\frac{1}{r}$ in ϵ

$$V_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \int \underline{r' \cos\theta' \rho(\vec{r}') d\vec{r}'}$$



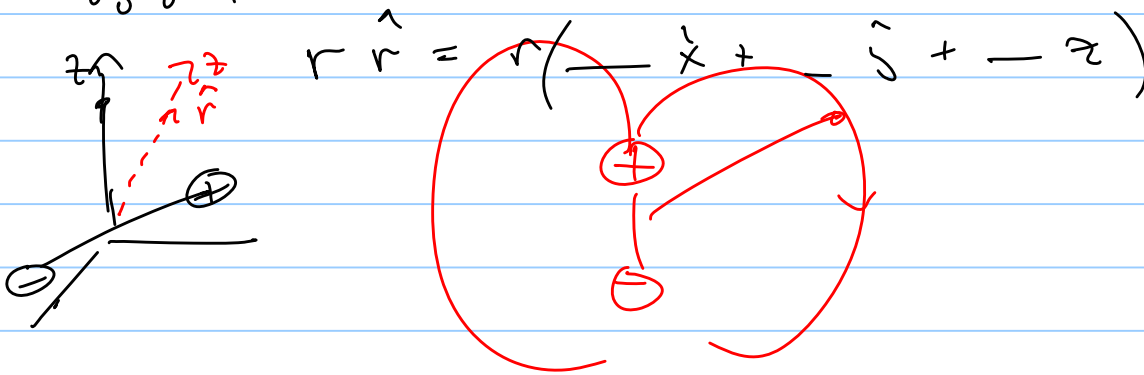
$$\begin{aligned} \underline{r \cdot r'} &= r' \cos\theta' = \hat{k} \cdot \vec{r}' = |\hat{k}| |\vec{r}'| \cos\theta' \\ V_{\text{dipole}} &= \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2} \\ \vec{p} &= \int \vec{r}' \rho(\vec{r}') d\vec{r}' \end{aligned}$$

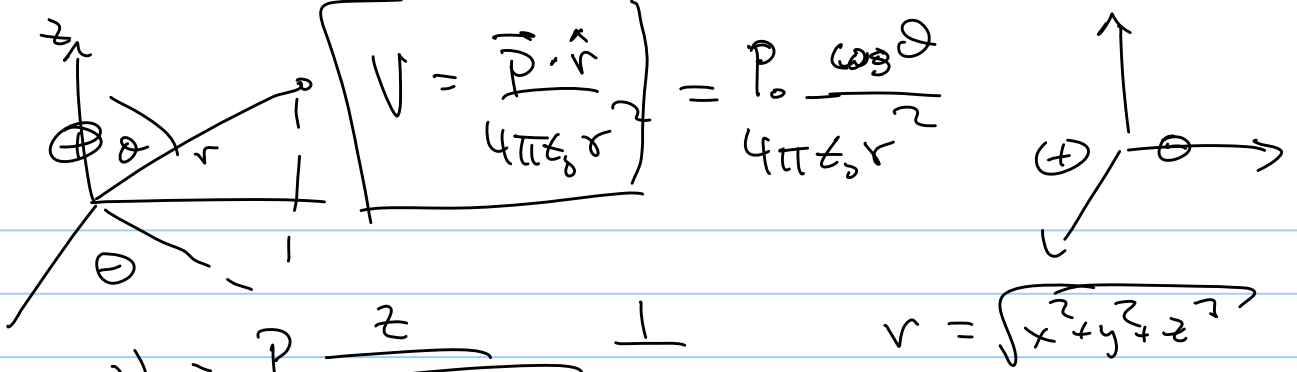


Write an integral expression for the dipole moment of the charge distribution $\rho = k(R-2r) \sin\theta$

R/r^2

$$\vec{p} = \int_0^R \int_0^{2\pi} \int_0^\pi \vec{r} k(R-2r) \sin\theta r^2 \sin\theta d\theta d\phi dr \frac{R}{r^2}$$





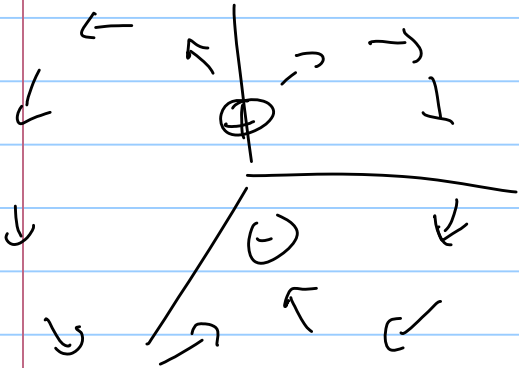
$$V = \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2} = \frac{p_0 \cos \theta}{4\pi\epsilon_0 r^2}$$

$$V(x, y, z) = \frac{p_0}{4\pi\epsilon_0} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \frac{1}{(x^2 + y^2 + z^2)^{3/2}}$$

$$r \cos \theta = z \quad \cos \theta = \frac{z}{r}$$

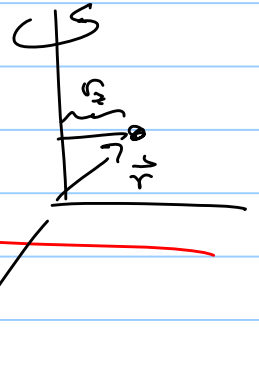
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\vec{E} = -\vec{\nabla} V(x, y, z) \Rightarrow E_z = -\frac{\partial V}{\partial z} \quad E_y = -\frac{\partial V}{\partial y} \quad E_x = -\frac{\partial V}{\partial x}$$

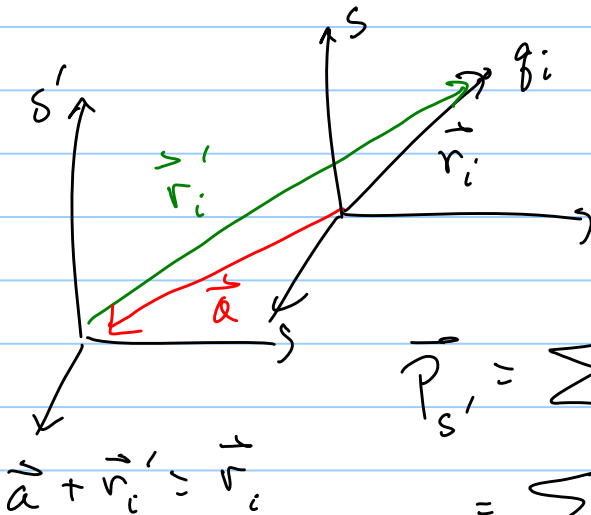


Have an expression for $V_{\text{dipole}} \neq \vec{E}_{\text{dipole}}$. Do these expressions depend on coord system chosen
moment of inertia:

$$I = \sum m_i r_i^2$$



Is dipole moment indep. of coords?



$$\vec{P}_S = \int \vec{r}' \rho(\vec{r}') d\tau = \sum_i \vec{r}'_i q_i$$

$$\sum_i \delta(\vec{r} - \vec{r}'_i) q_i$$

$$\vec{P}_{S'} = \sum_i q_i \vec{r}'_i = \sum_i q_i (\vec{r}_i - \vec{a})$$

$$= \sum_i q_i \vec{r}_i - \sum_i q_i \vec{a} = \vec{P}_S - \vec{a} \sum_i q_i$$

If $\sum_i q_i = 0$ (no net charge) then

$$\vec{P}_{s'} = \vec{P}_s$$