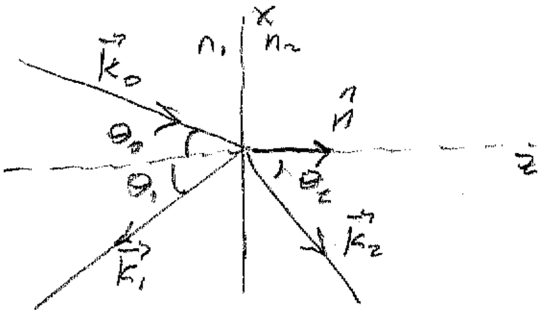
Reflection + Transmission : oblique angle.

first look at wave vectors (local $\vec{k}$ is like a "ray")



we will match tangential components of $\vec{E}, \vec{H}$

- all $\vec{k}$'s are in same plane (x-z here)
  "plane of incidence" POI
- $\vec{E}$ can be $\perp$ to POI → "S" polariz. "skim"
  $\parallel$ to POI → "p" polariz. "plunge"

in either case, B.C. say that

$\hat{n} \times (\vec{E}_0 + \vec{E}_1) = \hat{n} \times \vec{E}_2$ at $z=0$ (continuity in tang. E)

- matching amplitude components → Fresnel eq.

but this is only possible for all x if

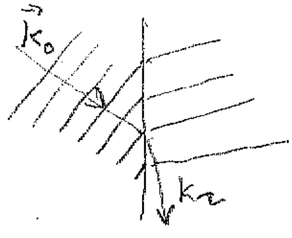
$$e^{i k_{0x} x} = e^{i k_{1x} x} = e^{i k_{2x} x} \quad \text{for all } x$$

$$\rightarrow n_0 \frac{\omega}{c} \sin\theta_0 = n_1 \frac{\omega}{c} \sin\theta_1 = n_2 \frac{\omega}{c} \sin\theta_2$$

principles phase is continuous along surface.

$n_0 = n_1 \rightarrow \theta_0 = \theta_1$ incid. angle = reflected.

and $n_1 \sin\theta_0 = n_2 \sin\theta_2$ Snell's Law.

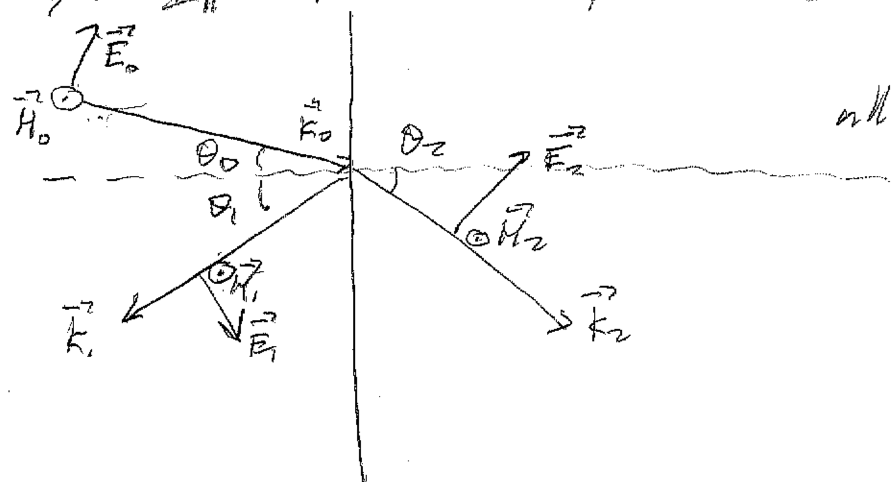


here $n_1 > n_2$ refr. away from normal

refraction is toward normal
if $n_1 < n_2$

Calculation of r, t

$E_\perp$ to plane of incidence is in book
here, do $E_\parallel$ to POI ("P" polarization)



match tangential E

$$E_0^0 \cos\theta_0 - E_1^0 \cos\theta_1 = E_2^0 \cos\theta_2$$

and H:

$$H_0^0 + H_1^0 = H_2^0$$

inside medium,

$$\nabla \times \vec{E} = -\frac{\mu}{c}\frac{\partial \vec{H}}{\partial t}$$

$$\vec{k} \times \vec{E} = \frac{\omega}{c}\vec{H} \qquad \mu \to 1$$

$$kE^0 = \frac{\omega}{c}H^0$$

$$k_0 E_0^0 + k_1 E_1^0 = k_2 E_2^0 \quad \text{or} \quad n_1 (E_0^0 + E_1^0) = n_2 E_2^0$$

$$E_0^0 \cos\theta_0 = E_1^0 \cos\theta_0 + \frac{n_1}{n_2}(E_0^0 + E_1^0)\cos\theta_2$$

$$\rightarrow E_1^0 = \underbrace{\frac{n_2 \cos\theta_0 - n_1 \cos\theta_2}{n_2 \cos\theta_0 + n_1 \cos\theta_2}}_{r_\parallel} E_0^0$$

$$= \frac{\tan(\theta_0 - \theta_2)}{\tan(\theta_0 + \theta_2)} E_0^0$$

$$E_2^0 = \frac{n_1}{n_2} E_0^0 (1 + r_{\parallel})$$

$$= \underbrace{\frac{2 n_1 \cos\theta_0}{n_2 \cos\theta_0 + n_1 \cos\theta_2}}_{t_{\parallel}} E_0^0$$

Brewster angle

at $\theta_0 = \theta_b$ $\quad r_{\parallel} \to 0$ $\qquad$ true at $\theta_0 + \theta_2 = \pi/2$

$$n_2 \cos\theta_0 = n_1 \cos\theta_2$$

use Snell's law:

$$n_1 \sin\theta_0 = n_2 \sin\theta_2$$

multiply

$$n_1 n_2 \sin\theta_0 \cos\theta_0 = n_1 n_2 \sin\theta_2 \cos\theta_2$$

$$\to \sin 2\theta_0 = \sin 2\theta_2$$

true if $\theta_0 = \theta_2$ $\quad$ (no $\quad n_0 \neq n_2$)

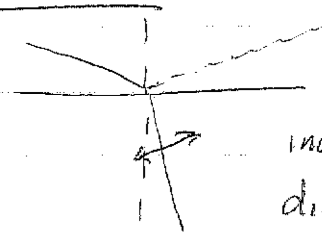
or $\pi - 2\theta_0 = 2\theta_2$

$$\to \underline{\theta_0 + \theta_2 = \pi/2}$$

$$n_2 \cos\theta_0 = n_1 \cos\left(\frac{\pi}{2} - \theta_0\right) = n_1 \sin\theta_0$$

$$\to \underline{\underline{\tan\theta_b = n_2/n_1}}$$

suppose $n_1 < n_2$



induced dipole radiate in direction of $\vec{E}_2$

if $\theta_1 + \theta_2 = \frac{\pi}{2}$ no radiation in direction of refl. wave.

Power refl + transm. coefficients, R, T
at normal incidence,

$$R = \frac{\langle S_1 \rangle}{\langle S_0 \rangle} = \frac{n_1 |E_1|^2}{n_1 |E_0|^2} = r^2$$

$$T = \frac{n_2}{n_1} t^2$$

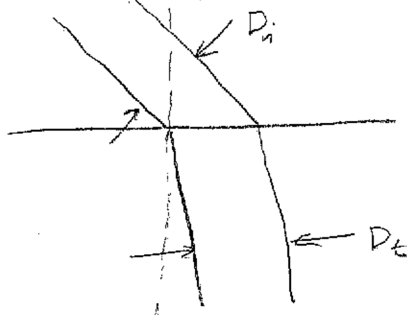
at an angle, consider power normal to surface.

$$R = \left| \frac{\langle \vec{S}_1 \rangle \cdot \hat{n}}{\langle \vec{S}_0 \rangle \cdot \hat{n}} \right| = r^2 \quad \text{still}$$

$$T = \frac{n_2 |E_2^0|^2}{n_1 |E_0^0|^2} \frac{\cos\theta_2}{\cos\theta_0}$$

with this correction $R + T = 1$

note that refracted beam size changes



for $n_1 < n_2$

$D_i < D_t$