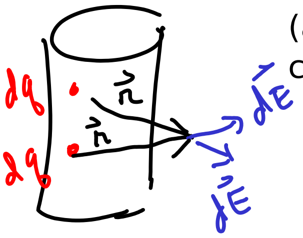
Exam 2 Feb. 7 **Name..**

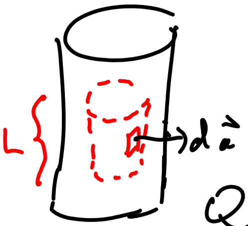
You need only derive integral expressions from which Mathematica will yield the answer. For example, simply writing the expression $\int \vec{B}\cdot d\vec{r}$ is insufficient yielding little credit.

1. A long cylinder of radius R_0 has uniform charge density ρ_0 distributed within. (a) In which direction does $\vec{E}$ point and why? (b) Derive an expression for the electric field inside the cylinder.



(a) Symmetrically located dq's generate dE's whose horizontal components cancel. E points radially. (3 pts)

(b) Use a cylindrical Gaussian surface inside to find E inside.

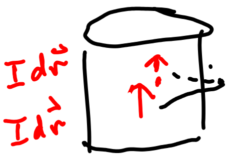


$$\oint \vec{E}\cdot d\vec{a} = \int |\vec{E}||d\vec{a}|\cos\phi = |\vec{E}|\int da = E\,2\pi r L \quad (3\text{pts})$$

E is the same for every tile so it comes outside integral.

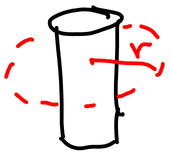
$$Q_{enclosed} = \int \rho\, dvol = \rho \pi r^2 L \;\therefore\; \vec{E} = \frac{\rho r}{2\epsilon_0}\hat{r} \quad (4\text{pts})$$

2. A current I is uniformly distributed on the surface of a long wire of radius R. The current moves only on the outside of the cylinder. (a) In which direction does B point and why? (b) Derive an expression for B outside the wire and justify your answer.



(a) Symmetrically located I dr's generate dB's pointing into the page. B points around the wire. (3 pts)

(b) Using ring Amperian path around wire.



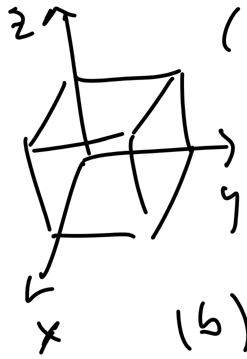
$$\oint \vec{B}\cdot d\vec{r} = \oint |\vec{B}||d\vec{r}|\cos\phi = B\oint dr = B\,2\pi r \quad (3\text{pts})$$

$$I_{enclosed} = \int \vec{J}\cdot d\vec{a} = I \qquad \oint \vec{B}\cdot d\vec{r} = \mu_0 I \qquad \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\varphi} \quad (4\text{pts})$$

3. Explain in a few sentences what you would do to check Stokes theorem for the lower surface (in the xy plane) of the next problem.

Integrate curl V dot da over lower surface area of the cube (5 pts). Perform a line integral of V over the 4 paths on the perimeter (5 pts) of that area. Check to see if the two answers are the same.

4. A unit cube sits in the first quadrant of a cartesian coordinate system with one vertex at the origin and the opposite vertex at $(1, 1, 1)$. (a) Derive an expression for the volume integral in the diverence theorem (don't evaluate it) for $\vec{V} = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$. (b) Derive an expression for the flux (don't evaluate it) through the lower surface (in the xy plane) of the cube.



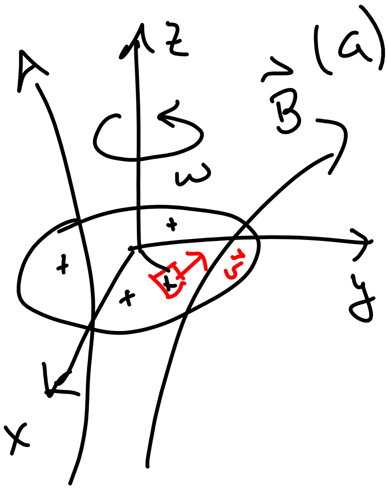
(a) $\int \vec{\nabla}\cdot\vec{V}\,dx\,dy\,dz = \int_0^1\int_0^1\int_0^1 \left(\frac{\partial}{\partial x}y^2 + \frac{\partial}{\partial y}(2xy+z^2) + \frac{\partial}{\partial z}(2yz)\right)dx\,dy\,dz$

(5 pts)

(b) $\int \vec{V}\cdot d\vec{a} = \int \left(y^2\hat{x} + (2xy+z^2)\hat{y} + 2yz\hat{z}\right)\cdot dx\,dy(-\hat{z})$

$= -\int_0^1\int_0^1 2yz\,dx\,dy$ (5 pts)

5. (a) Find the force on a charged record of radius r_0 with angular rotation rate $\omega_0\hat{z}$ located symmetrically in the xy plane in the presence of a magnetic field $\vec{B}(x, y, z)$. The charge density is $\sigma = \sigma_0$. (b) If the axis of rotation moves from being through the center of the record to the edge of the record (both axes of rotation are parallel to each other) how would your answer change?



(a) $\int d\vec{F} = \int \vec{K}\times\vec{B}\,da$ (3 pts) : $\vec{K} = \sigma\vec{v} = \sigma_0\omega r\hat{\varphi}$ (3 pts)

$\hat{\varphi} = -\sin\varphi'\hat{x} + \cos\varphi'\hat{y}$

$$\vec{K}\times\vec{B} = \sigma\omega r \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\varphi & \cos\varphi & 0 \\ B_x & B_y & B_z \end{vmatrix}$$

limits: φ $0 \to 2\pi$ (2 pts)
r $0 \to r_0$

$da = r\,d\varphi\,dr$ (3 pts)

3 pts discretionary

(b) Magneto statics is not valid since in a given region the charge density varies with time.

(6 pts)