Nonlinear Optics
Homework 3
due Wednesday, 21 Feb 2011

- **Problem 1: $LiNiO_3$ pockels cell**
  The lithium niobate crystal has a different set of nonlinear coefficients in its electro-optic tensor from KDP, which was our example in class. It can work with a voltage that is transverse to the propagation direction of the beam.

- **a. Set up the 3x6 electro-optic tensor for LiNi03 and calculate the three different index ellipsoid equations for E_x, E_y and E_z input polarization directions. The equivalent expression for KDP is equation 11.3.2.**

- **b. For the general case, including all three field components, write out the full 3x3 η tensor that is implied by the first two terms of 11.2.11. Note that the field that goes into this tensor is the applied DC field. What you get is complicated, but normally, the field is applied in a specific x, y or z direction.**

- **c. The easiest orientation is where the DC field is applied in the X direction and the beam propagates in the Z direction. Write the new tensor for this situation. To do the full problem, we would find the new coordinate system that diagonalizes this tensor. It is simplest to work under the approximation that we can ignore the r42 term. When you set r42=0, the matrix is now very similar to what we had for KDP, and can be easily diagonalized.**

- **d. Following the reasoning in the book for KDP, find an expression for the half-wave voltage for the LiNiO3 crystal in this configuration. Explain why in this case the Pockels cell can be run at much lower voltage if constructed properly.**

- **For further information: the voltage can also be applied in the z direction, with the wave propagating in the x direction. This allows access to the high r33 coefficient. The trouble is that there is a lot of static birefringence, so a second, passive crystal oriented at 90o must also be put in-line to compensate the static birefringence of the first (kind of like we did for the zero-order waveplate).**

- **Problem 2: Quasi phase matching**
  Boyd problem 2.9

- **Problem 3:**
  Following the calculations done for second harmonic generation shown in the notebook mixing solutions.nb, numerically solve for the intensity vs. propagation length for sum frequency mixing.
  a) First start wih the nonlinear coupled equations 2.2.10, 2.2.12a, and 2.2.12b and redefine the fields scaled to the total intensity $I_{tot} = I_1 + I_2 + I_3$, along the lines of what is shown in 2.7.13. It is easier to express the fields in terms of a single complex variable a_i instead of $u_i\, e^{i\,\phi_i}$(which is the treatment in the book). Unlike the case for SHG, there will be some residual frequency dependence to the scaling factor. Choose your normalization factor to include the frequency ω_3, then the expressions to normalize A_1 and A_2 will have an extra factor of ω_1 / ω_3 and ω_2 / ω_3, respectively. Note that eqn 2.7.19 should have the $\epsilon_0\, c$ term in the numerator of the square root.

  b) Construct the numerical solution using NDSolve[ ] as shown in my example.
  Find input conditions to generate plots like Figures 2.6.2, 2.6.3 and 2.8.2.

  c) Note that the case of difference frequency mixing (DFM) is also solved here - it is just a matter of what input waves are there. Show an example for DFM, illustrating an OPA (strong pump at ω_3 and a weak seed at ω_1. (This is an extension of the plot 2.8.2 that

includes saturation.)

d) For sum-frequency generation, determine input conditions that allow complete conversion of the two inputs to ω_3 without any back conversion. (Hint: consider the Manley-Rowe relations.)

- **Problem 4: Numerical solutions of quasi phase matching**
  Plot the growth of signal for second-harmonic generation in a quasi-phasematched crystal in which the d_{eff} is modulated sinusoidally. Make plots for two cases, 1st order and 3rd order quasi-phase matching.

- **Problem 5: second-harmonic generation (non-depleted case)**
  Boyd problem 2.19