Lecture 16

Shadowitz: 4-2 capacitance, 4-3 electrostatic energy

Next exam March 7

I will set up an InkSurvey for those who got a score <50% on exam 2.

Communicate with me individually about your score or misunderstandings on exam 2.

You get InkSurvey credit for an honest attempt at solution and when use ID number.

REVIEW: charging a capacitor and discharging it through a watermellon.

Questions:

http://www.youtube.com/watch?v=gj1pkyCL75E

incongruous: How did they fake this? Did they put dynamite in the mellon?

congruous: How do I calculate the effect knowing the energy stored in the caps?

Can I estimate the initial energy stored in the cap by measuring how far the pieces flew?

http://www.youtube.com/watch?v=2K0cEX9ex3U

modifying: Would the same thing happen to a balloon of water?

generalizing/analogy: How similar is this process to that using dynamite?

How would this differ if a high energy laser hit the watermellon?

causal/creative: Is the reason for the explosion vaporizing water due to the transfer of electrical energy into heat energy?

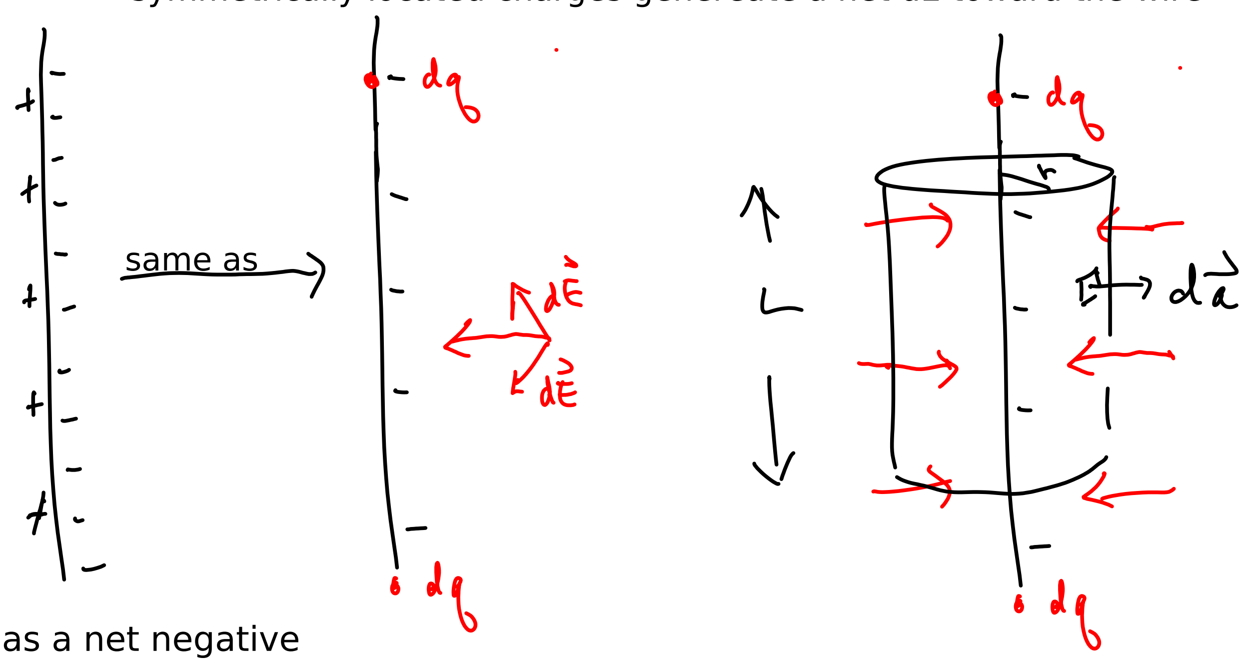
informational: How much energy was stored in the caps before discharge?

How can I estimate the KE of the bits after the explosion?

How far did the bits fly away?

symmetrically located charges genereate a net dE toward the wire

InkSurvey:



This has a net negative charge density

This is a model for current flowing in a wire, not how real current flows. It is used to illustrate length contraction in a simple way, not model realistic current flow.

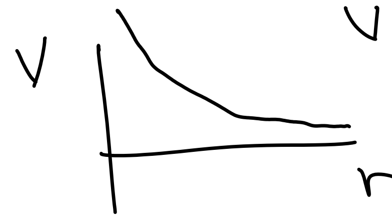
Gauss's law for the cylinder

$$\oint \vec{E}\cdot d\vec{a} = \oint |\vec{E}||d\vec{a}|\cos\theta = E\oint da = E\,2\pi r L$$

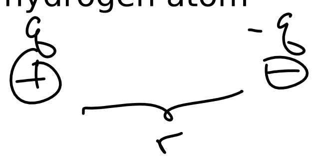
$$\frac{Q_{encl}}{\epsilon_0} = \int_0^L \frac{\lambda\,dz}{\epsilon_0} = (\lambda_- - \lambda_+)\frac{L}{\epsilon_0}$$

To plot PE=qV first write V for proton

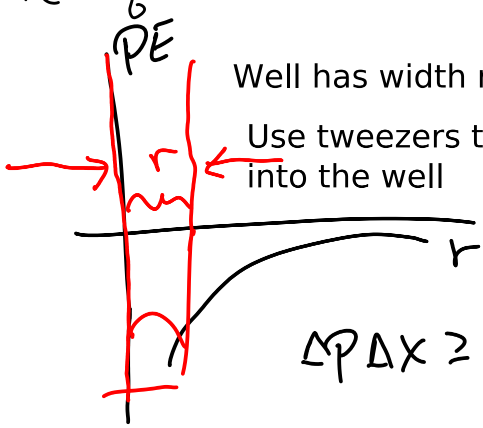
$$V = \frac{q}{4\pi\epsilon_0 r}$$



InkSurvey: hydrogen atom

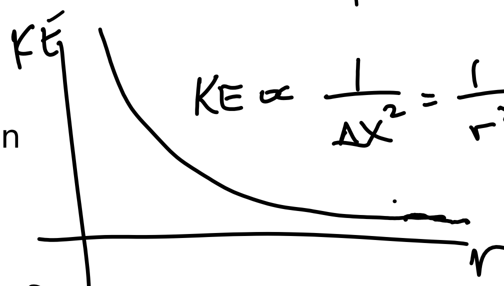


$PE = -q\Delta V$



Well has width r

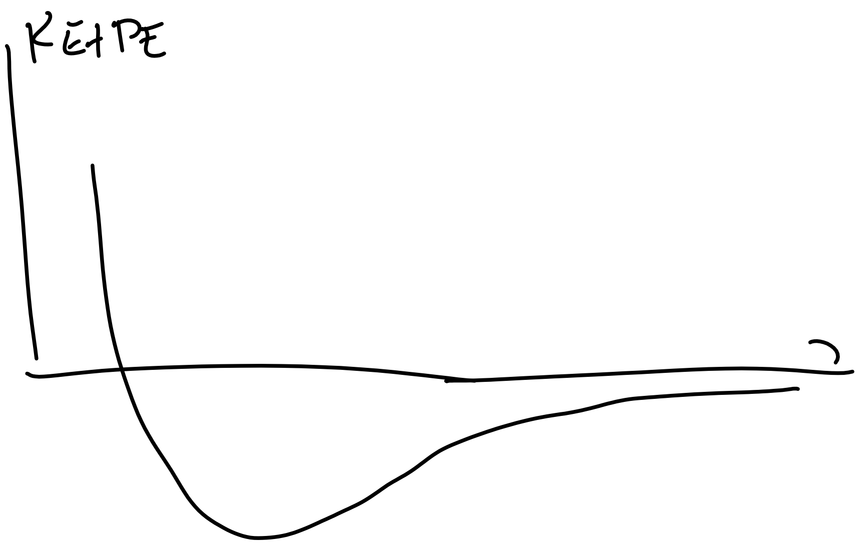
Use tweezers to drop electron into the well



$$KE \propto \frac{1}{\Delta x^2} = \frac{1}{r^2}$$

$$\Delta p \Delta x \geq \frac{\hbar}{2} \qquad KE = \frac{\Delta p^2}{2m} = \frac{\hbar}{2}\left(\frac{\hbar}{2\Delta x}\right)^2 \frac{1}{2m} \qquad \Delta x \simeq r$$

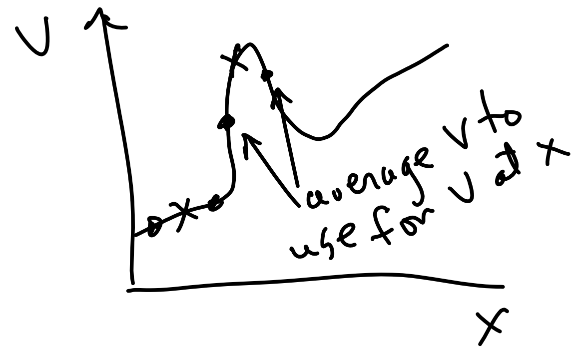
Model the electron as being in a potential well of width r. I move it in at constant speed to a distance r and drop it into the well.



muddiest points:

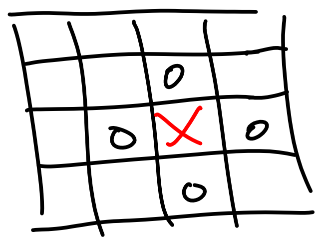
-relaxation method to Laplaces eqn.

$$\nabla^2 V = 0 \qquad \text{1-D} \qquad \frac{d^2 V}{dx^2} = 0$$

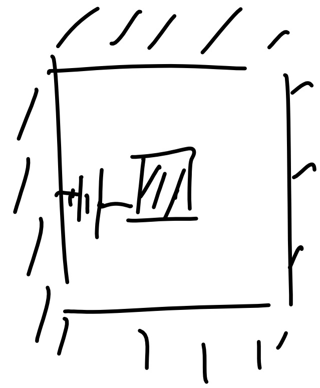


Maxima and minima occur only at the boundaries.

Write a formula for a cell inside boundaries which averages the 4 nearby cell values.



$$V_x = \frac{V_0^{up} + V_0^{down} + V_0^{left} + V_0^{right}}{4}$$



-definition of capacitance

$$C = \frac{Q}{\Delta V}$$

Need ΔV to calculate C

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$E \pi r^2 = \frac{\sigma \pi r^2}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

$\vec{E}$

r

d

$$|\Delta V| = \left| -\int_0^d \vec{E} \cdot d\vec{r} \right|$$

$$\Delta V = E d$$

$$Q_{on\ cap} = \sigma \, Area_{cap}$$

$$C = \frac{\sigma \, Area_{cap}}{E d} = \frac{\sigma \, Area_{cap}}{\frac{\sigma}{\epsilon_0} d} = \frac{A}{\epsilon_0 d}$$

-Why am I teaching you QM and not E&M?

Most people remember the material better if they see the connections one topic has with others.

-when to use Gauss's law

When you need to find E and the charge distribution is symmetric so you know the direction of E. The latter allows you to choose a Gaussian surface that is easy to calculate the flux from.

You can also use Gauss's law to find the charge distribution given E (see 1st hmwk problem)

-How to apply Poisson's eqn.?

$$\nabla^2 V = -\rho/\epsilon_0$$

We will spend a lot of time with conductors with no charge density between them.
That reduces this eqn to Laplaces equation.
The first example of a solution to Laplace's eqn is the relaxation problem.

-How is the energy in the field related to the energy required to build the charge distribution?

See below.

-How does B induce E?

This will be covered later.

-What is Gauss's law?

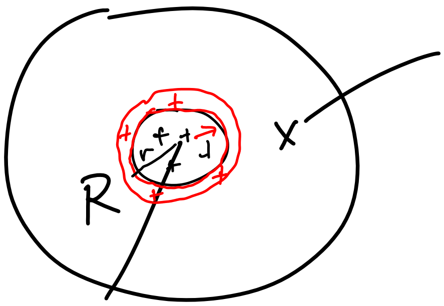
$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

- Why am I a grumpy sort of guy?

We need to cover a lot of material. If someone wants to move the lecture topic in another direction and we don't have time to discuss that please come see me after class rather than expecting the class time to become devoted to that topic.

How do I calculate the work if it is easy to determine the voltage at every point in the charge distribution due to all the charges that are present?

This voltage is NOT the voltage due to only the charges that have been brought it!!!



$$V = -\int_{\infty}^{r} \vec{E}_{tot} \cdot d\vec{r}$$

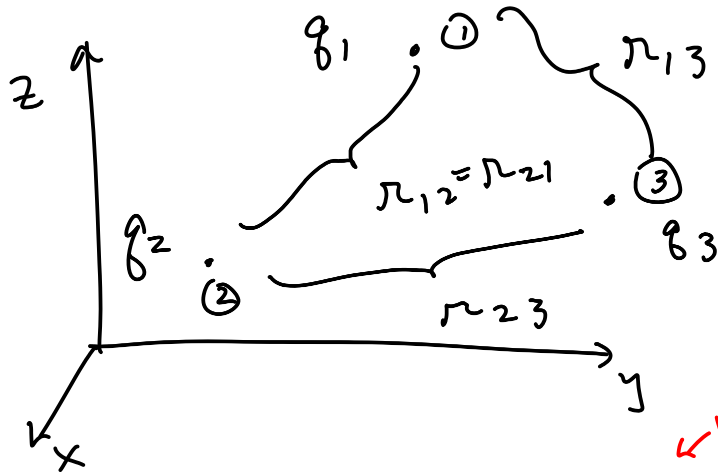
Questions

How do I calculate the work required given the voltage at each point in the charge distribution due to all charges present (congruous)?

What simple example can I use to understand how this is done (modifying)?

Choose 3 charges and calculate the work required to bring them in individually. From this result see if a formula can be obtained which uses the voltage at each charge while all other charges are present.

Use a three charges to find work (two is too simple and four too complicated).



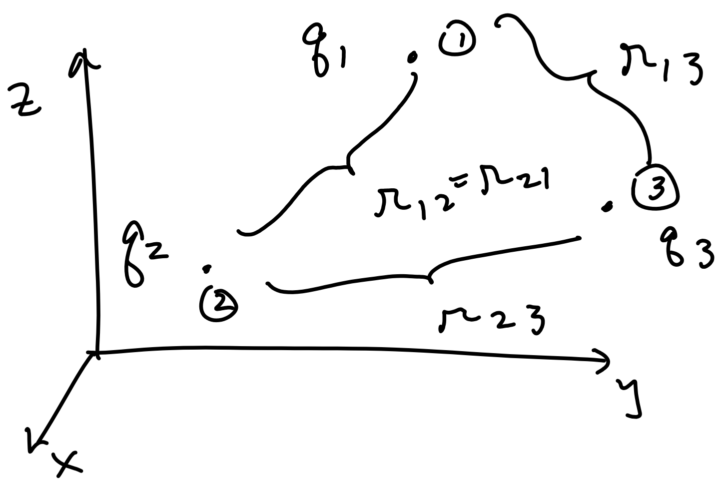
now me

Let charge 1 be brought in first, then charge 2, then charge 3.

$$W_{me} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \underbrace{\frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}} \right)$$

What is the voltage at each charge due to all others (informational)?

This the voltage we want to use in our new formula.



$$V_1 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}} \right)$$ This is the voltage at 1 due to charges 2 and 3

$$V_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{12}} + \frac{q_3}{r_{23}} \right)$$ This is the voltage at 2 due to charges 1 and 3

$$V_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right)$$ This is the voltage at 3 due to charges 1 and 2

$$W_{me} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$ This is the work I do in bringing the charges in individually.

Multiply this by 2

$$2W_{me} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left[\underbrace{q_1\left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}}\right)}_{V(1)} + \underbrace{q_2\left(\frac{q_3}{r_{23}} + \frac{q_1}{r_{12}}\right)}_{V(2)} + \underbrace{q_3\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)}_{V(3)}\right]$$

Notice how terms can be grouped to make V(1) and V(2) and V(3).

$$W_{me} = \frac{1}{2}\sum_{i=1}^{N} q_i\, V(P_i)$$

i^{th} point

Questions

$$W = \frac{1}{2}\int V\, dq$$

Integration by parts

$$\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$$

$$\int_a^b \frac{d}{dx}(fg)\,dx = \int_a^b f\frac{dg}{dx}\,dx + \int_a^b g\frac{df}{dx}\,dx$$

$$\parallel$$

$$fg\Big|_a^b$$

$$\int_a^b f\frac{dg}{dx}\,dx = -\int_a^b g\frac{df}{dx}\,dx + fg\Big|_a^b$$

You can transfer the derivative from g to f at the cost of a minus sign and boundary term.

Use the divergence theorem to illustrate integration by parts in vector calculus

$$\int \vec{\nabla}\cdot(f\vec{A})\,d\tau = \int f(\vec{\nabla}\cdot\vec{A})\,d\tau + \int \vec{A}\cdot\vec{\nabla}f\,d\tau$$

$\downarrow$ divergence theorem

$\oint f\vec{A}\cdot d\vec{a}$ is the boundary term

$$W = \frac{1}{2}\int V\,dq = \frac{1}{2}\int V\rho\,d\tau$$

substitute

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$W = \frac{\epsilon_0}{2}\int(\vec{\nabla}\cdot\vec{E})\,V\,d\tau$$

$$\int \vec{\nabla}\cdot(f\vec{A})\,d\tau = \int f(\vec{\nabla}\cdot\vec{A})\,d\tau + \int \vec{A}\cdot\vec{\nabla}f\,d\tau$$

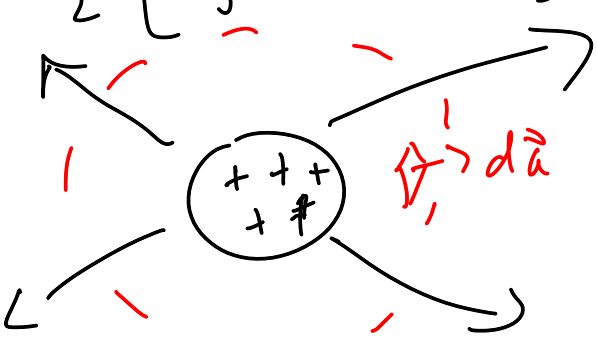
$\downarrow$ divergence theorem

$\oint f\vec{A}\cdot d\vec{a}$ is the boundary term

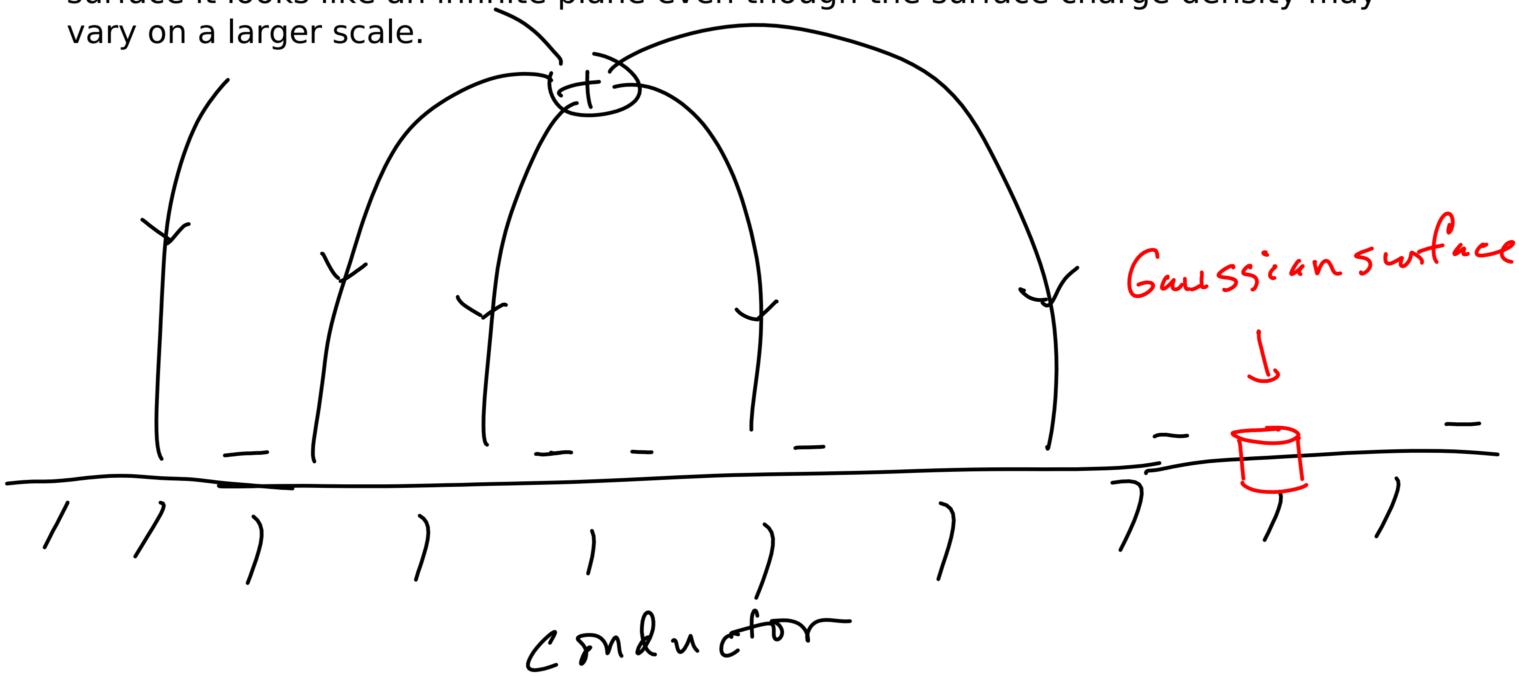
Transfer the derivative from E to V

$$W = \frac{\epsilon_0}{2}\left[-\int \vec{E}\cdot\underbrace{\vec{\nabla}V}_{=\,-\vec{E}}\,d\tau + \oint V\vec{E}\cdot d\vec{a}\right]$$

$$W = \frac{\epsilon_0}{2}\left[\int E^2\,d\tau + \oint V\vec{E}\cdot d\vec{a}\right]$$



For problem 1 use Gauss's law to find the charge density on the surface of a metal given E. On a metal E is perpendicular to the surface. If you go close enough to the surface it looks like an infinite plane even though the surface charge density may vary on a larger scale.



Let this Gaussian surface be small wrt variations in E and have one end cap in the conductor and the other end cap just above the surface.

$$\oint \vec{E}\cdot d\vec{a} = \underbrace{\int |\overset{0}{\vec{E}}||d\vec{a}|\cos\phi}_{\text{lower end cap}} + \underbrace{\int |\vec{E}||d\vec{a}|\cos\phi}_{\text{upper end cap}} + \underbrace{\int |\vec{E}||d\vec{a}|\overset{0}{\cos 90^\circ}}_{\text{body}}$$

same over cap

$$= \underbrace{\int |\vec{E}||d\vec{a}|}_{\text{upper end cap}} = E(\text{area cap}) = \int \frac{\sigma\, da}{\epsilon_0} = \frac{\sigma\, \text{areacap}}{\epsilon_0}$$

$$\Rightarrow \quad E = \frac{\sigma}{\epsilon_0}$$

If we are given E then this eqn yields the surface charge density. Since E will vary over the surface so will the charge density.