2a) Differential equations of the form $X'' = B_i^2 X$

have solutions of the form $e^{\pm B_i x}$, where B_i might be real or imaginary depending on the sign of B_i^2. So solutions might include things like

$$\boxed{e^{kx},\ e^{-kx},\ e^{ikx},\ e^{-ikx}}$$

note $\Longrightarrow$ where $k = |B_i|$

We also know that linear combinations of these are solutions, and appropriate combos of real exponentials form hyperbolic trig functions, while combos of complex exponentials yield regular trig functions, so we might also use:

$$\boxed{\cosh(kx),\ \sinh(kx),\ \cos(kx),\ \sin(kx)}$$

and their counterparts in y and z.

b) Let's think about z first. At $z=0$, $V(x,y,0) = V_0$.

Then, V should drop monotonically to zero as $z \to \infty$. After all, very far away, there are grounded plates all around us, and no sources to be seen.

sin & cos are periodic, so they won't be much help

$\sinh(z)$ & $\cosh(z)$ & e^{z} diverge as z increases, so they're even less help. Only functions of the form $\boxed{e^{-k_3 z}}$

can possibly yield a legitimate solution in z.

What about in x? We need $V=0$ at $x=0$ and $x=W$.

Getting zeros in two places with one of those places being the origin is something that only sine functions can deliver.

Same argument holds in y.

Thus X must go like $\sin(k_1 x)$ and Y like $\cos(k_2 y)$

A solution of the form $e^{-k_3 z}$ comes from a positive K_3^2,

and sines come from negative K_1^2 and K_2^2, so our choices are consistent with the requirement that

$K_1^2 + K_2^2 + K_3^2$ needs to be able to be zero.

And it <u>will</u> be zero as long as $k_1^2 + k_2^2 + k_3^2 = 0$, so we only have to find two of those and we'll automatically know the third.

c) Our full series solution should therefore look like:

$$V(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{mn} e^{-k_{3mn} z} \sin(k_{1n} x)\sin(k_{2m} y)$$

Since only two of k_1, k_2, and k_3 are actually independent, you only need a double sum. Each value of n yields a new k_1, and likewise for k_2 and m, but once you pick those, k_3 is set in stone. No third degree of freedom.