MATH 348 - Advanced Engineering Mathematics
Homework 9, Spring 2008

LINEAR ALGEBRA - ROW REDUCTION AND SOLUTIONS TO LINEAR SYSTEMS

1. Given the linear system

$$
\begin{aligned}
6x_1 + 18x_2 - 4x_3 &= 20 \\
-x_1 - 3x_2 + 8x_3 &= 4 \\
5x_1 + 15x_2 - 9x_3 &= 11.
\end{aligned}
$$

   Determine the general solution to the linear system and describe this set geometrically.

2. Given the following augmented matrix

$$
\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array}\right].
$$

   Determine h and k such that the corresponding linear system is :

   (a) consistent with a unique solution.
   (b) consistent with infinitely many solutions.
   (c) inconsistent.

3. Determine if $\mathbf{b}$ is a linear combination of the vectors formed from the columns of the matrix $\mathbf{A}$.

$$
\mathbf{A} = \left[\begin{array}{cc} 5 & 3 \\ -4 & 7 \\ 9 & -2 \end{array}\right], \quad \mathbf{b} = \left[\begin{array}{c} 22 \\ 20 \\ 15 \end{array}\right]
$$

4. Determine the values of h for which the vectors are linearly dependent.

$$
\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ -1 \\ -3 \end{array}\right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -5 \\ 7 \\ 8 \end{array}\right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 1 \\ h \end{array}\right]
$$

5. Given,

$$
\mathbf{A} = \left[\begin{array}{ccc} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{array}\right], \quad \mathbf{w} = \left[\begin{array}{c} 2 \\ 1 \\ -2 \end{array}\right].
$$

   (a) Is $\mathbf{w}$ in the column space of $\mathbf{A}$? That is, does $\mathbf{w} \in \mathrm{Col}\ \mathbf{A}$?
   (b) Is $\mathbf{w}$ in the null space of $\mathbf{A}$? That is, does $\mathbf{w} \in \mathrm{Nul}\ \mathbf{A}$?