MATH 225 - Differential Equations w/ Honors May 12, 2008
Homework 1, Field 2008 **Due**: May 13, 2008

INTEGRATION REVIEW - INTRODUCTION TO DIFFERENTIAL EQUATIONS

1. Evaluate the following integrals:
   (a) $\int x^3 \cos(5x)dx$
   (b) $\int x^2 \sin(2x^3)dx$
   (c) $\int \frac{4-2x}{(x^2+1)(x-1)^2}dx$
   (d) $\int \frac{5x}{3x-1}dx$

   Hint: For (a)-(b) you must decide to use integration-by-parts or substitution. The fraction in (c) can be made into a simpler sum by partial fractions. For (d) consider a substitution.

2. Assuming that $s \in \mathbb{R}$ evaluate the following improper integrals:
   (a) $\int_0^\infty t^3 e^{\beta t} e^{-sx} dt$, where $\beta \in \mathbb{R}$ and $s > \beta$.
   (b) $\int_0^\infty e^{-st} \cos(\omega t)dt$, where $\omega \in \mathbb{R}$ and $s > 0$.
   (c) $\int_0^\infty e^{-st} \sin(\omega t)dt$, where $\omega \in \mathbb{R}$ and $s > 0$.

   Hint: For (b),(c), perform two steps of integration-by-parts and watch for the original integral to reappear.

3. Solve the following equations for the variable x:
   (a) $\ln(x) - \ln(x-4) = -13$
   (b) $\ln(x) + \ln(x-4) = -13$
   (c) $e^{2(\ln(x)-\ln(x^2))} = 1$

4. Consider the first order linear homogenous ordinary differential equation(ODE), $\frac{dy}{dt} = (1+t)y$. We can think of ODEs as equations which define the slope of the solution $y(t)$ for a given point (t, y).
   (a) Using HPGSOLVER plot the vector field associated with the ODE.
   (b) Using HPGSOLVER plot solution curves passing through the points $(t_0, y_0) = \{(-1,2), (-1,-2), (-1,0), (-\frac{1}{2},3), (-\frac{1}{2},-3), (-2,2), (-2,-2), (1,1), (-1,1)\}$.
   (c) Verify that $y(t) = Ce^{\frac{1}{2}t^2+t}$ satisfies the ODE regardless of the choice of the constant C.

5. Find all possible solutions to the following differential equations:
   (a) $y' = 0$
   (b) $y' = y$
   (c) $y'' = 0$
   (d) $y'' = y$
   (e) $y'' = -y$

   Hint: For these problems you are not expected to use techniques or methods explored in the text or in class. You need to instead think about the question that the differential equation is asking. Considering question (b), we are being asked what is the function y that has the property that its first derivative is equal to itself.