

$$\frac{1}{(1+\epsilon)^{1/2}} \approx \left(1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{4} + \dots\right)$$

?

Note Title

2/19/2007

$$V = \frac{q}{4\pi\epsilon_0} - \frac{1}{r(1+\epsilon)^{1/2}}$$

$$\epsilon = \frac{a^2}{r^2} - \frac{2a}{r} \cos\theta$$

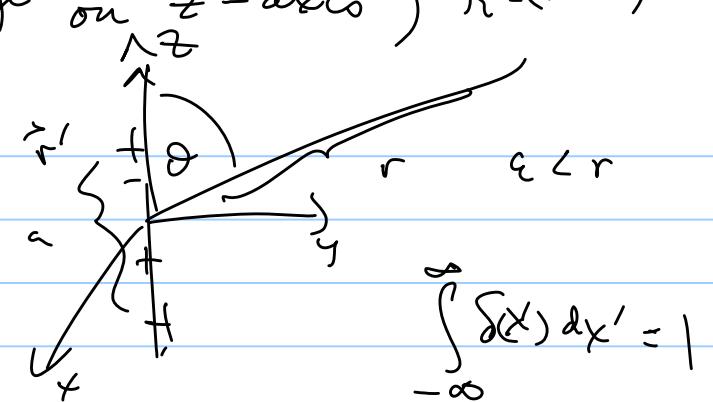
$$\left(\frac{1}{r}\right)^{1/2} = \frac{1}{r(1+\epsilon)^{1/2}} = \frac{1}{r} \left[1 - \frac{1}{2} \left(\frac{a^2}{r^2} - \frac{2a}{r} \cos\theta \right) + \frac{3}{8} \left(\frac{a^2}{r^2} - \frac{2a}{r} \cos\theta \right)^2 + \dots \right]$$

$$= \frac{1}{r} \left[1 + \left(\frac{a}{r}\right) \cos\theta + \left(\frac{a}{r}\right)^2 \left[\frac{3 \cos^2\theta - 1}{2} \right] + \dots \right]$$

$$= \frac{1}{r} \sum_{l=0}^{\infty} \left(\frac{a}{r}\right)^l P_l(\cos\theta) \quad r > a$$

- Axial multipole moments (charge on \hat{z} -axis) $\vec{r} = (\vec{r} - \vec{r}')$

$$\rho(\vec{r}') = \delta(x') \delta(y') \lambda(z')$$



$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r') d^3 r'}{r} = \frac{1}{4\pi\epsilon_0} \iiint \frac{\delta(x') \delta(y') \lambda(z') dz'}{(r^2 + z'^2 - 2rz \cos\theta)^{1/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \int \lambda(z') \sum_{l=0}^{\infty} \left(\frac{z'}{r}\right)^l P_l(\cos\theta) dz'$$

$$= \frac{1}{4\pi\epsilon_0} \sum_{l=0}^{\infty} \frac{M_l P_l(\cos\theta)}{r^{l+1}}$$

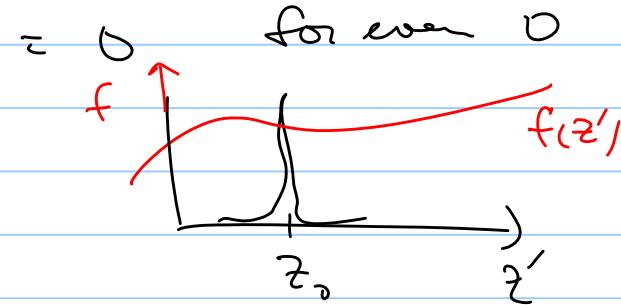
axial multipole moment

$$\text{where } M_l = \int \lambda(z') (z')^l dz'$$

Calculate the monopole and dipole moment for the charge distribution $\lambda(z') = q_0 \delta(z' - a/2) - q_0 \delta(z' + a/2)$.

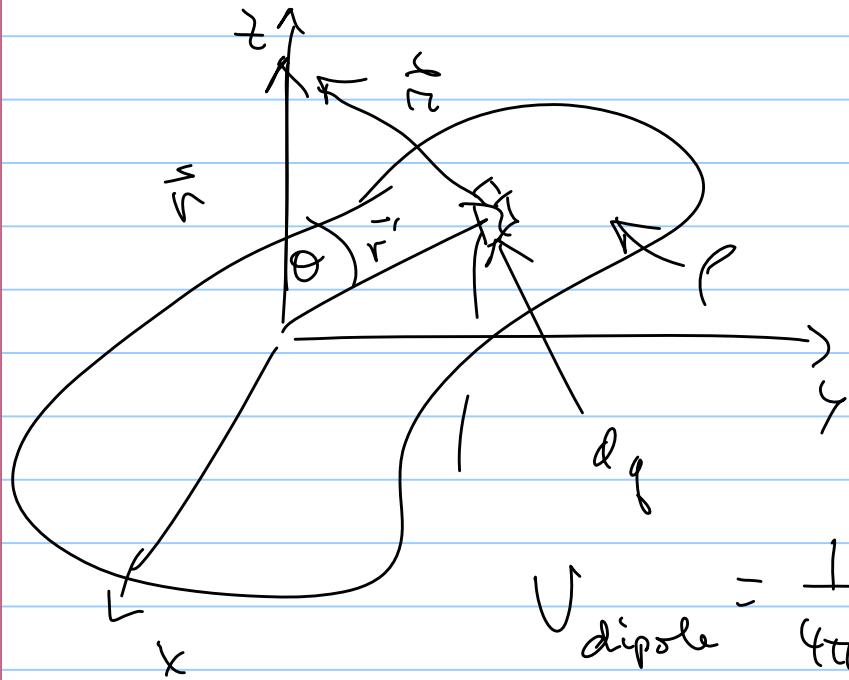
$$\begin{aligned}
 M_\ell &= \int \left\{ g_0 \underbrace{\delta(z' - \frac{a}{2})}_{z' = \frac{a}{2}} - g_0 \underbrace{\delta(z' + \frac{a}{2})}_{z' = -\frac{a}{2}} \right\} z'^\ell dz' \\
 &= g_0 \left(\frac{a}{2}\right)^\ell - g_0 \left(-\frac{a}{2}\right)^\ell = \left(\frac{a}{2}\right)^\ell 2g_0 \text{ for } \ell = 1, 3, 5, \dots
 \end{aligned}$$

$$\int_{-\infty}^{\infty} \delta(z_0 - z') f(z') dz' = f(z_0)$$



In general $V(r, \theta, \varphi) = \frac{\sqrt{4\pi}}{4\pi g_0} \sum_{\ell, m} M_{\ell, m} \frac{Y_{\ell m}(\theta, \varphi)}{r^{\ell+1}}$

$$M_{lm} = \sqrt{4\pi} \int p(\vec{r}') (r')^l Y_{lm}(\theta', \phi') d^3 r'$$



find V only on z axis

$$V(z)$$

$$V_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \int r' \cos\theta' \hat{p} \cdot \hat{r}' d^3 r'$$

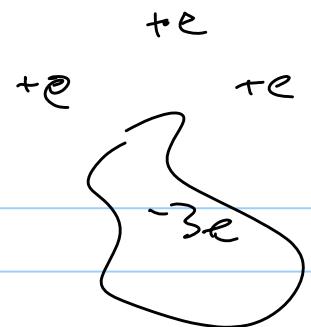
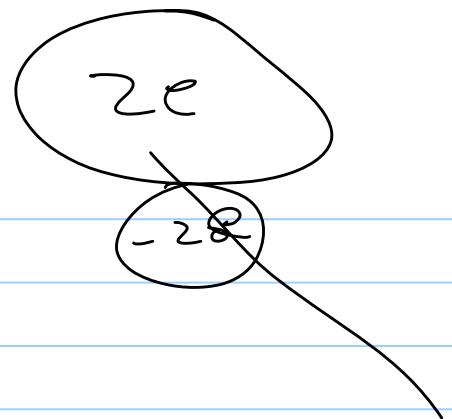
$$\vec{P} = \int \vec{r}' \hat{p} \cdot d^3 r'$$

dipole mom

$$\vec{p}(\vec{r}') = \sum_i \delta^3(\vec{r}' - \vec{r}_i) q_i$$

$\sum_i q_i \vec{r}_i = \vec{P}$

if p continuous



A hand-drawn diagram illustrating vector addition. It shows two vectors originating from the same point: \vec{r}_1 pointing along the horizontal axis, and \vec{r}_2 pointing diagonally upwards and to the right. The resultant vector \vec{P} is shown as a diagonal line segment extending from the origin to the tip of \vec{r}_2 . The equation $q_1 \vec{r}_1 + q_2 \vec{r}_2 = \vec{P}$ is written below the diagram.

$$q_1 \vec{r}_1 + q_2 \vec{r}_2 = \vec{P}$$