

RESPONSE TO FEEDBACK FROM HOMEWORK 3 - QUESTION 5

1. Personal Recap - At this point we will pause our work with Fourier series and thus it is a good time to summarize our recent developments. Please do the following for EACH of the sections 11.1, 11.2, 11.3, 11.4:

- (a) In roughly one paragraph summarize the section. Pretend you are trying to explain what was missed to a friend of yours who missed class. Your goal should be to recount key concepts and connect them to the important mathematics encountered.¹
 - (b) List one to three questions remaining with the material.
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Q: When does $a_0 = 0$?

A: $a_0 = 0$ can happen when the periodic function is odd or when $\int_{-\pi}^{\pi} f(x)dx = 0$ for a 2π -periodic function.

Q: When do you use a half-wave rectifier?

A: <http://en.wikipedia.org/wiki/Rectifier> - The first paragraph gives great examples.

Q: When would you choose an even extension?

A: If you have the option of periodically extending a function then you might as well extend the function in a symmetric way. If this is the case then it is unclear whether to choose an odd or even extension. If you want the extended function to be smooth then you should choose an even extension. However, this requires two integral evaluations. If you are not concerned about smoothness then odd extensions will work as well.

Q: When would you use a complex Fourier series?

A: There is no choice mechanism for which you would use, real or complex, otherwise only one would be discussed in books and class. The choice mechanism basically runs like this. If you want to simplify your coefficient calculations and are not concerned with having imaginary numbers then you would use a complex FS. If you do not want complex numbers then you would use a real FS.

Q: How do you convert a complex FS to a real FS.

A: The process has three parts. First, rationalize your c_n coefficients. After this write the doubly infinite sum as two infinite sums one over the positive integers and one over the negative integers. Finally, join the two sums. If everything has been done correctly then all imaginary terms drop out.

Q: Can you go from a real FS to complex FS?

A: Yes, we did the derivation in class. The derivation is also done in 11.4 of the text.

Q: Why use a FS to estimate a function when we know the function?

A: This is a great question. Say you are given a function $f(x) = x$ which is 2π -periodic on the interval $-\pi < x < \pi$. Well, what is $f(100)$? This is difficult to answer in the current form of f however, if you knew the FS then you could determine this value, within some precision, pretty easily from the FS of f .

¹Please be concise. The key is to strip the material down to what you feel is essential. If you write too much then you are likely to forget much of the summary.

Q: How is this useful to civil engineers?

A: This is a good question and comes up all the time. Not being a civil engineer I wouldn't know and this is maybe something you should ask a civil engineer in your department. I can say that structural engineers should be interested in periodic forcing of structures. In particular an important question is whether a given structure will resonate when forced periodically. Question 1 from HW5 addresses this question from the standpoint of simple mass spring systems. If possible resonance should be avoided either by changes to the structure or by avoiding long term exposure to periodic forcing involving harmonics, which at the systems resonant frequencies.

Q: How are Fourier transforms used in the 'real-world'?

A: Fourier transforms are ubiquitous and are used in many areas of applied science. In general, any time you want to break a function down into amplitude and frequency information. For functions of a continuous variable you would use the continuous Fourier transform and for functions of a discrete variable you would use the discrete Fourier transform. In the continuous case you might want to transform a partial differential equation into Fourier space and solve the problem there. In the discrete case you might want to take a discrete Fourier transform of a data set that some apparatus outputs.